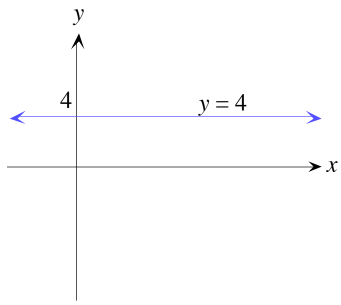
Answers for Math 19 Exercises   Waner

1. The Cartesian Plane and Distance

1.



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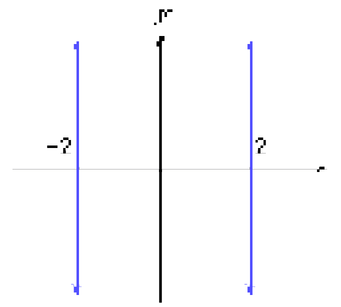

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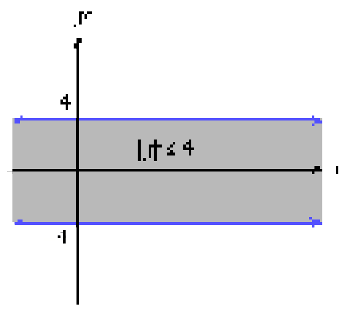

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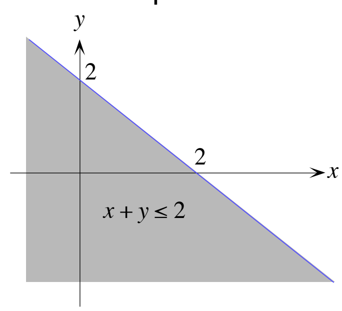
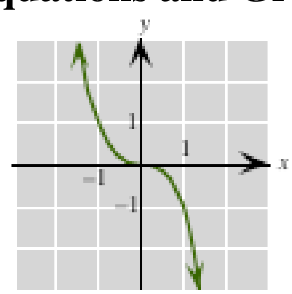


5. $\sqrt{2}$ 6. $\sqrt{26}$ 7. $\sqrt{a^2+b^2}$ 8. $\sqrt{2}\,|b-a|$ **9**. 1/2   10. $-1/2$ **11**. The perpendicular bisector of the line joining those two points.

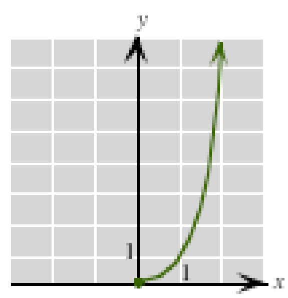
2. Equations and Graphs

1.



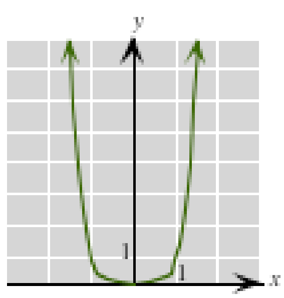
-x^3

2.



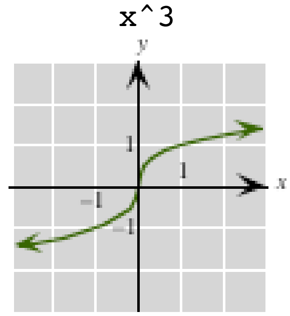
x^3

3.



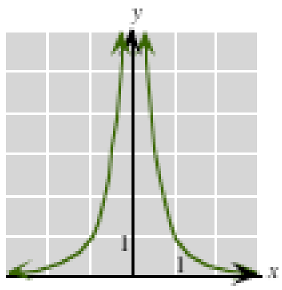
x^4

4.



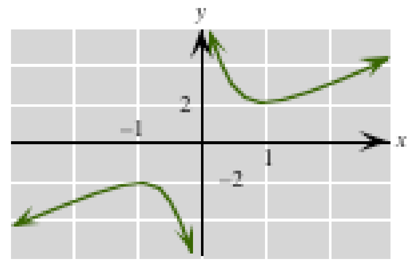
x^(1/3)

5.



1/x^2

6.



x+1/x

7. Circle with center (2,−1) and radius 3 **8**. Circle with center (−3,1) and radius 2 **9**. Sphere with center (3, −1) and radius 3 **10**. The single point $(\frac{1}{2}, \frac{1}{3})$ **11**. The empty set **12.** The upper semicircle with center (0, 1) and radius 3. **13**. The left semicircle with center (1, −4) with radius 5 **14**. $(x + \frac{1}{2})^2 + (y + \frac{1}{2})^2 = \frac{1}{2}$ **15**. $x = -\sqrt{9 - y^2}$ **16**. $y = -2 + \sqrt{8 - (x-1)^2}$

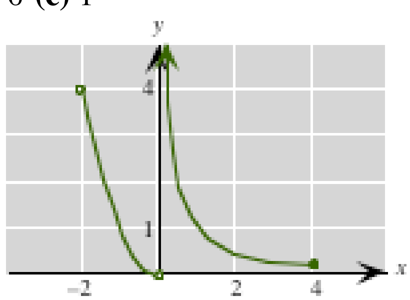
3. Straight Lines

1. **2.** 3. −2 **4.** Undefined **5.** .5 **6.** $/\sqrt{3}$ **7.** −2 **8.** $(d-b)/(c-a)$ **9.** 0 **10.** $y = 3x$ **11.** $y = \frac{1}{4}x - 1$ **12.** $y = 10x - 203.5$ **13.** $y = -5x + 6$ **14.** $y = -x/8 + 3/4$ **15.** $y = \frac{3}{4}x - \frac{5}{4}$ **16.** $y = \frac{x}{\sqrt{2}} - \frac{1}{2\sqrt{2}} + 5$ **17.** $y = -\frac{1}{3}(x - \frac{1}{3})$ **18.(a)** 2.5 ft/sec **(b)** 20 feet along the track **(c)** after 6 seconds **19.(a)** 130 miles per hour **(b)** $s = 130t - 1300$ **20.** $T = 1.8C + 32$; 86˚F; 72˚F; 14˚F; 7˚F **21.** $T(r) = (1/4)r + 45$; $T(100) = 70$°F

4. Functions and Their Graphs

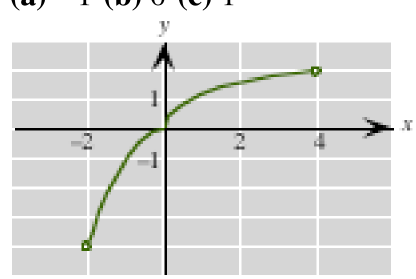
1.(a) 2 **(b)** 0 **(c)** 65/4 **(d)** x^2+1/x **(e)** $(s+h)^2$ + 1/(s+h) **(f)** $(s+h)^2$ + 1/(s+h) - (s^2 + 1/sÛ) **2. (a)** 1 **(b)** 1 **(c)** 0 **(d)** 27 **3.(a)** h(2x+h) **(b)** 2x+h **4.(a)** -h(2x+h) **(b)** -(2x+h) **5.(a)** h(2x+h+1) **(b)** 2x+h+1

6. **(a)** 1 **(b)** 0 **(c)** 1



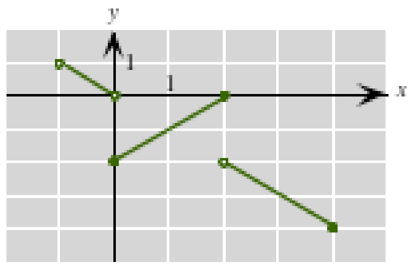
(x^2)*(x<=0)+(1/x)*(0<x)

7. **(a)** −1 **(b)** 0 **(c)** 1



(−1*x^2)*(x<=0)+SQRT(x)*(x>0)

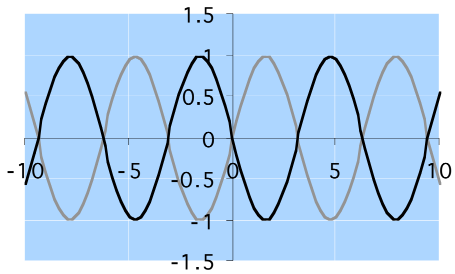
8. **(a)** −2 **(b)** −1 **(c)** 0 **(d)** −3



(-x)*(x<0)+(x-2)*(0<=x)*(x<=2)+(-x)*(2<x)

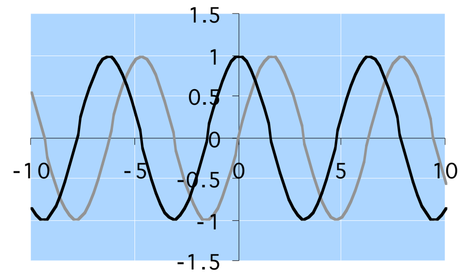
5. Trigonometric Functions

1.



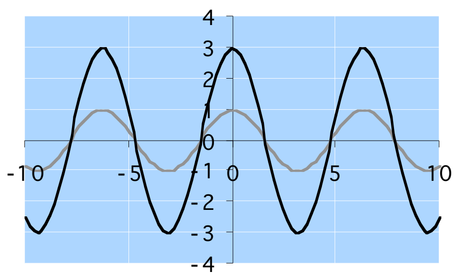
sin*x* is shown in grey

2.



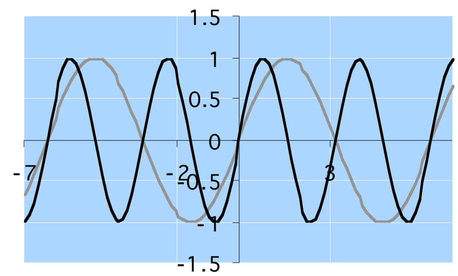
sin*x* is shown in grey

3.



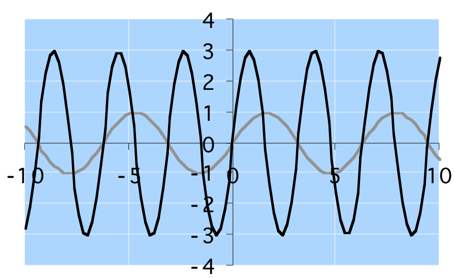
cos*x* is shown in grey.

4.



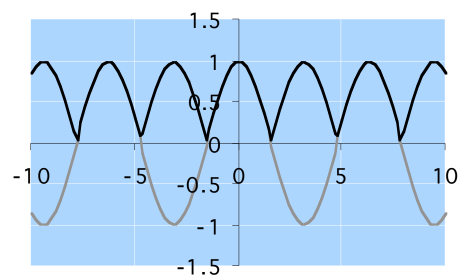
sin*x* is shown in grey

5.



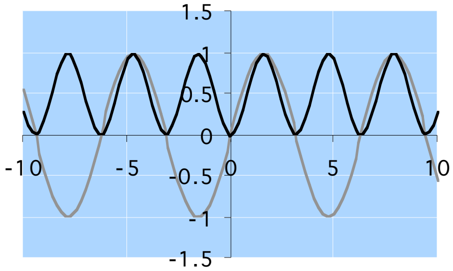
sin*x* is shown in grey

6.



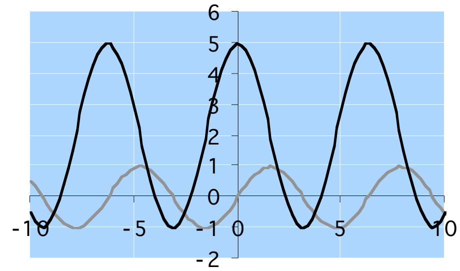
cos*x* is shown in grey

7.



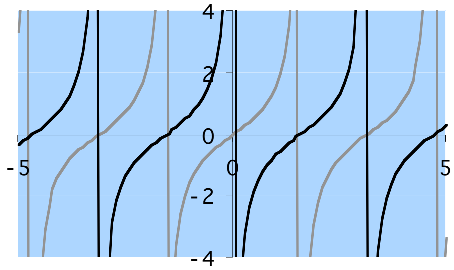
sin*x* is shown in grey

8.



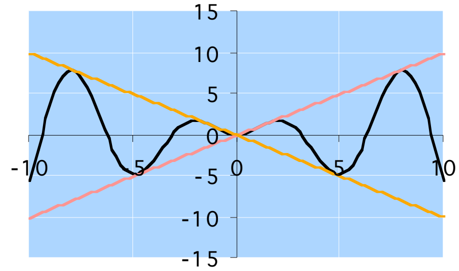
sin*x* is shown in grey

9.

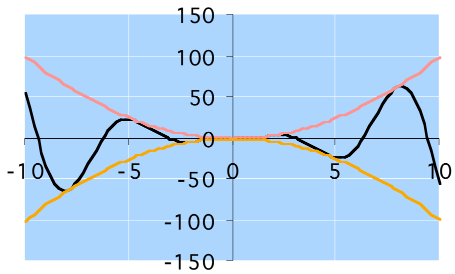


tan*x* is shown in grey

10.



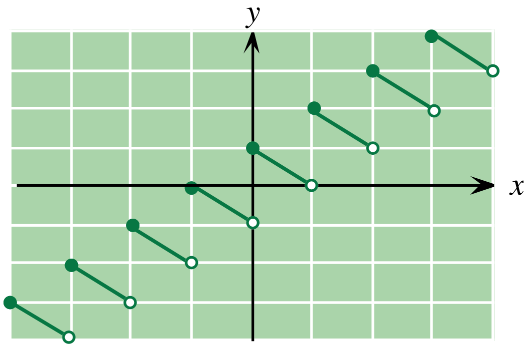
11.



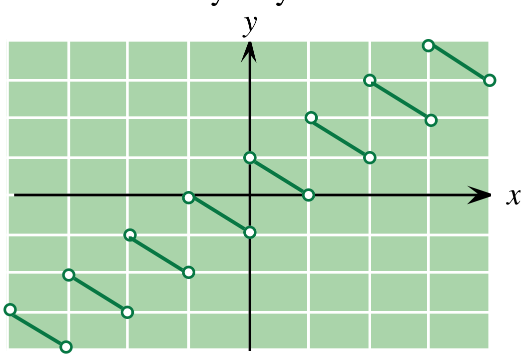
6. Limits Intuitively & Graphically

Estimate the limits in Exercises 1–14 numerically.

1. 4 **2.** Does not exist **3.** Does not exist **4.** 0 **5.** 3/2 **6.** 2 **7.** Diverges to $+\infty$ **8.** □ **9.** 0 **10.** Diverges to $+\infty$ **11.** 0 **12.** 0 **13.** □ **14.** □ **15.(a)** 2 **(b)** 1 **(c)** 0 **(d)** $+\infty$ **16.(a)** 1 **(b)** 1 **(c)** -1 **(d)** $-\infty$ **17.(a)** 1 **(b)** 1 **(c)** 2 **(d)** dne **(e)** 1 **(f)** 2 **18.(a)** -1 **(b)** -3 **(c)** 2 **(d)** dne **(e)** 2 **(f)** 0 **19.(a)** 1 **(b)** $+\infty$ **(c)** $+\infty$ **(d)** $+\infty$ **(e)** not defined **(f)** -1 **20.(a)** 0 **(b)** $+\infty$ **(c)** $-\infty$ **(d)** dne **(e)** not defined **(f)** -2 **21.(a)** -1 **(b)** $+\infty$ **(c)** $-\infty$ **(d)** dne **(e)** 2 **(f)** 1 **22. (a)** $+\infty$ **(b)** 0 **(c)** $+\infty$ **(d)** 0 **(e)** 0 **(f)** 1 **23.** It is possible for $\lim_{x \to a} f(x)$ to exist even though $f(a)$ is not defined. An example is $\lim_{x \to 1} \frac{x^2-3x+2}{x-1}$. **24.** If $f(a)$ is defined, then the statement is only true when the function f is continuous at $x = a$. **25.** It could, by fluctuating at levels that increase without bound. **26.** An example is $f(x) = (x-1)(x-2)$. **27.** Continuous on its domain **28.** Discontinuous at $x = 0$ **29.** Discontinuous at $x = 1$ **30.** Discontinuous at $x = -1$ **31.** Continuous on its domain **32.** Continuous on its domain **33.** Discontinuous at $x = -1$ and 0 **34.** Discontinuous at $x = 0$ and 1 **35.(A), (B), (D), (E)** **36.** □ **37.** □ **38.** □1 **39.** No value possible **40.** No value possible **41.** □/3 **42.** -1 **43.** No value possible **44.** Continuous on its domain **45** Continuous on its domain **46** Continuous on its domain **47.** Continuous on its domain **48.** Discontinuity at $x = 0$ **49.** Continuous on its domain **50.** Discontinuity at $x = 0$ **51.** Discontinuity at $x = 1$ **52.** Not unless the domain of the function consists of all real numbers. (It is impossible for a function to be continuous at points not in its domain.) For example, $f(x) = 1/x$ is continuous on its domain—the set of nonzero real numbers—but not at $x = 0$. **53.** False. For example, $f(x) = 1/x$ is continuous on its domain, but its graph is not a single continuous curve. **54.** True. If the graph of a function has a break in its graph at any point a, then it cannot be continuous at the point a. **55.** Answers may vary. $f(x) = \sqrt{x}$ is such a function, since -1 is not in the domain of f. (For a function to be discontinuous at $x = a$, that point must be in the domain of f.) **56.** Answers may vary.



57. Answers may vary.



7. Rules for Limits

1. [illegible] **2.** -5 **3.** 6 **4.** $-1/2$ **5.** 4 **6.** -5 **7.** 0 **8.** 1/2 **9.** 0 **10.** -1 **11.** 12 **12.** -12 **13.** Diverges to $+\infty$ **14.** Diverges to $-\infty$ **15.** None; left and right limits differ **16.** Diverges to $-\infty$ **17.** 6 **18.** 1/4 **19.** 3/2 **20.** 2 **21.** Diverges to $+\infty$ **22.** 0 **23.** 3/2 **24.** $-2/3$ **25.** Diverges to $-\infty$ **26.** $\sqrt{2}/2$ **27.** 0 **28.** Diverges to $-\infty$

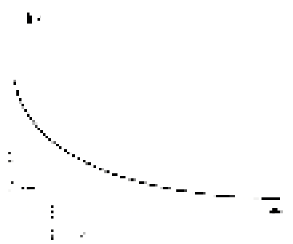
8. Intro to Calculus

1. $4a$, 12. **2.** $2a+1$, -1 **3.** $-1/a^2$, -1 **4.** $2a-4$, -6 **5.** 1/2 **6.** 0 **7.(a)** R **(b)** Q **(c)** P **8.(a)** P **(b)** Q **(c)** R **9. (a)** (1, 0) **(b)** None **(c)** (−2, 1) **10. (a)** (0, 1), (2, −0.5) **(b)** (−1, 0), (3, 0) **(c)** (1, 0) **11.** $(a, f(a))$; $f'(a)$. **12.** secant; the points $(a, f(a))$ and $(a+h, f(a+h))$. **13. (a)** (B) **(b)** (B) **(c)** (C)

9. Derivatives

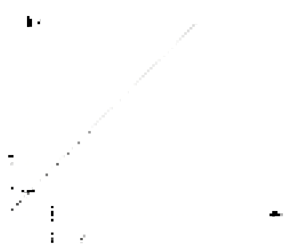
1. 3 **2.** $4x + 1$ **3.** $2 - 2x$ **4.** $1 - 6x^2$ **5.** m **6.** $-2/x^2$ **7.** $-2/x^3$ **8.** $-1/x^2$ **9.** $-1/(x+2)^2$ **10.** $\frac{1}{2\sqrt{x-1}}$ **11.** $2ax$ **12.** $\frac{1}{3x^{2/3}}$ **13.(a)** $f'(1) = 2$ **(b)** Not differentiable at 0 **14.(a)** Not differentiable at 1 **(b)** Not differentiable at 0 **15.** Not differentiable at 0; $f'(1) = 1/3$ **17.** $2x$ **18.** By the time we reach the end of the computation, we have canceled the offending h in the denominator, so that putting $h = 0$ makes sense. What is happening is this: The difference quotient is not defined when $h = 0$, but, after doing some algebra and canceling the h we change it to an expression that *is* defined when $h = 0$. **19.**(C) **20.**(E) **21.**(A) **22.**(D) **23.**(F) **24.**(B) **25.(a)** −96 ft/sec **(b)** −128 ft/sec **26.(a)** (B) **(b)** (B) **(c)** (A) **(d)** 1992 **(e)** 0.05; in 1996, the total number of state prisoners was increasing at a rate of approximately 50,000 prisoners per year. **27.** The tangent to the graph is horizontal at that point, and so the graph is almost horizontal near that point.

28.



29.

30..



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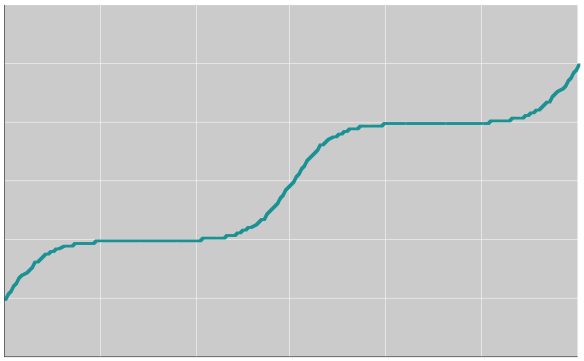

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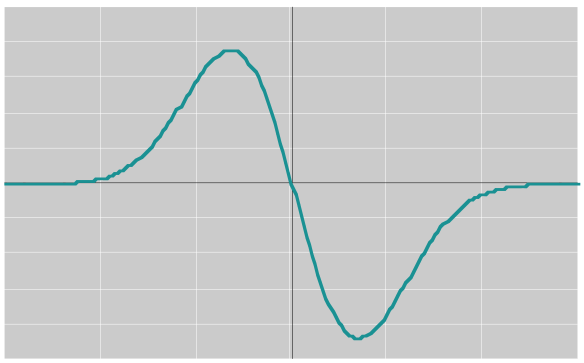
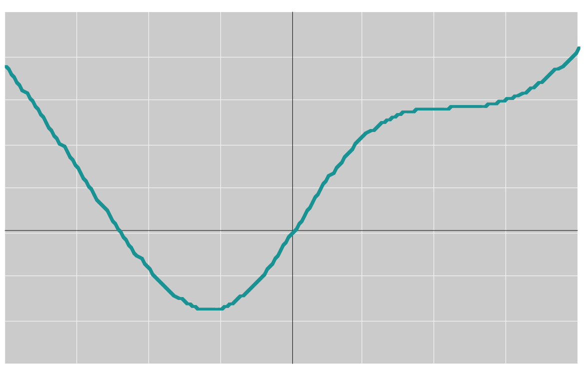


33.



10. Rules of Differentiation: Powers, Sums, Product, and Quotients

1. $5x^4$ **2.** $-4x^{-3}$ **3.** $-0.25x^{-0.75}$ **4.** $8x^3+9x^2$ **5.** $-1/x^2 - 2/x^3$

6. $\frac{dy}{dx} = 10(0) = 0$ (constant multiple and power rule)

7. $\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(x)$ (sum rule) $= 2x + 1$ (power rule)

8. $\frac{d}{dx}(4x^{-1} - 2x - 10) = \frac{d}{dx}(4x^{-1}) - \frac{d}{dx}(2x) - \frac{d}{dx}10$ (differences)

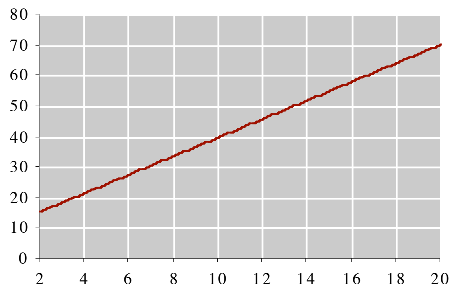
$= 4\frac{d}{dx}(x^{-1}) - 2\frac{d}{dx}x - 10\frac{d}{dx}1$ (constant multiples) $= -4x^{-2} - 2$ (power rule)

9. $\frac{dy}{dx} = \frac{d}{dx}(x^{3/2} + x^{5/2}) = \frac{d}{dx}(x^{3/2}) + \frac{d}{dx}(x^{5/2})$ (sums) $= \frac{3}{2}x^{1/2} + \frac{5}{2}x^{3/2}$ (power rule)

10. $f'(x) = 2x - 3$ **11.** $f'(x) = 1 + 0.5x^{-0.5}$ **12.** $g'(x) = -2x^{-3} + 3x^{-2}$ **13.** $g'(x) = -\frac{1}{x^2} + \frac{2}{x^3}$ **14.** $h'(x) = -\frac{0.8}{x^{1.4}}$ **15.** $h'(x) = -\frac{2}{x^3} - \frac{6}{x^4}$ **16.** $r'(x) = \frac{2}{3} - \frac{0.1}{2x^{0.9}} - \frac{4.4}{3x^{2.1}}$ **17.** $r'(x) = |x|/x - 1/x^2$ **18.** $s'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$ **19.** $t'(x) = 1$ **20.** $2.6x^{0.3} + 1.2x^{-2.2}$ **21.** $1.2(1-|x|/x)$ **22.** $2at + b$ **23.** $0.4x^{0.2} - 0.45x^{-0.1}$ **24.** $-\frac{2.2}{t^{2.1}} - \frac{1.2}{t^{2.2}}$

25. $8\pi r$ **26.** -32 **27.** $1/3$ **28.** $-3/16$ **29.** $y = 0$ **30.** $y = -2x + 3$ **31.** $x = -1/6$

32. No such values **33.** $x = 1/4$ **34.** **(a)** $s'(t) = 3.04t + 9.45$



(b) 1994–1995 ($t = 14$)

35. $P'(t) = -5.2t + 13$; increasing at a rate of 2.6% per year

36. **(a)** $s'(t) = -32t$; 0, −32, −64, −96, −128 ft/sec **b)** 5 seconds; downward at 160 ft/sec

37. After graphing the curve $y = 3x^2$, draw the line passing through $(-1, 3)$ with slope −6.

38. The slopes are the same.

39. $g'(x) = -f'(x)$

40. The left-hand side is not equal to the right-hand side. The *derivative* of the left-hand side is equal to the right-hand side, so your friend should have written $\frac{d}{dx}(3x^4+11x^5) = 12x^3 + 55x^4$.

41. We cannot apply the power rule to terms in the denominator. To take the derivative of $3/4x^2$, first write it in exponent form as $\frac{3}{4}x^{-2}$ and then use the power rule to get $-\frac{3}{2}x^{-3}$ or $-\frac{3}{2x^3}$.

42. The derivative of a constant times a function is the constant times the derivative of the function, so that $f'(x) = (2)(2x) = 4x$. Your enemy mistakenly computed the *derivative* of the constant times the derivative of the function. (The derivative of a product of two functions is not the product of the derivative of the two functions. The rule for taking the derivative of a product is discussed in the next chapter.).

43. A possible response is that the power rule applies only to function that happens to be given by a constant power of x. Since there certainly are functions that have nothing to do with constant powers of x, the power rule is very limited, and we need the general definition to deal with other kinds of functions.

Product & Quotient Rule

44. $6x(2x+1) + 3x^2(2) = 18x^2+6x$

45. $5x^4(1-x) + x^5(-1) = 5x^4-6x^5$

46. $4(4x-1) + (4x-1)(4) = 8(4x-1)$

47. $2x\sqrt{x} + \dfrac{x^2}{2\sqrt{x}} = 2x\sqrt{x} + \dfrac{x\sqrt{x}}{2} = \dfrac{5x\sqrt{x}}{2}$

48. $(8x+1)(x-x^2) + (4x^2+x)(1-2x) = -16x^3+9x^2+2x$

49. $(0.7x^{-0.3}-4)(x^{-1}+x^{-2}) + (x^{0.7}-4x-5)(-x^{-2}-2x^{-3})$

50. $2(2x^{0.5}- x^2)(x^{-0.5}-2x)$

51. $(0.3x^{1.1} - 4.2/x^{3.1})(7x-1) + 7(x^{2.1}/7 + 2/x^{2.1})$

52. $(x^2-3)(2x^2+1) + 2x^2(2x^2+1) + 4x^2(x^2-3)$

53. $1.1(x^{2.1}-x)(3.4-x^{-2.1}) + (1.1x+4)(2.1x^{1.1}-1)(3.4-x^{-2.1}) + 2.1x^{-3.1}(1.1x+4)(x^{2.1}-x)$

54. $\left(8x - \dfrac{1}{2\sqrt{x}}\right)\left(\sqrt{x} - \dfrac{2}{x^2}\right) + \left(4x^2 - \sqrt{x}\right)\left(\dfrac{1}{2\sqrt{x}} + \dfrac{4}{x^3}\right)$

55. $\dfrac{3(2x+4) - 2(3x-9)}{(2x+4)^2} = 30/(2x+4)^2$

56. $\dfrac{(6x-9)(2x+4) - 2(3x^2-9x+11)}{(2x+4)^2} = (6x^2+24x-58)/(2x+4)^2$

57. $\dfrac{\frac{1}{2}x^{-1/2}(x^{1/2}+1) - \frac{1}{2}x^{-1/2}(x^{1/2}-1)}{(x^{1/2}+1)^2} = \dfrac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}$

58. $-2/x^3$

59. $\dfrac{(x-5)(x-4) - x[(x-4) + (x-5)]}{[(x-5)(x-4)]^2} = (-x^2+20)/[(x-5)(x-4)]^2$

60. $\dfrac{3(x-5)(x-4)(x-1)-(3x-1)(x-4)(x-1)-(3x-1)(x-5)(x-1)-(3x-1)(x-5)(x-4)}{(x-5)^2(x-4)^2(x-1)^2}$

61. 64 **62.** 2 **63**. -3 **64**. 2

65. $y = 7x-3$ **66.** $y = x/64 + 11/16$ **67.** $y = 1/2$

68. The analysis is suspect, as it seems to be asserting that the annual increase in revenue, which we can think of as dR/dt, is the product of the annual increases, dp/dt in price, and dq/dt in sales. However, since $R = pq$, the product rule implies that dR/dt is not the product of dp/dt and dq/dt, but is instead

$$\frac{dR}{dt} = \frac{dp}{dt}\cdot q + p\cdot\frac{dq}{dt}.$$

11. Limits & Derivatives of Trig Functions

1. **(a)** 2/3 **(b)** Does not exist **(c)** 2 **(d)** 4/9 **(e)** 0 **(f)** 1/2 **(g)** 0 **(h)** 0 **(i)** $-\infty$ **(j)** 0
(k) 0 **(l)** 1/2 **(n)** 6 **(o)** $\pi/180$

2. **(a)** All real numbers **(b)** Yes

3. $\sec^2 x - \cos x$

4. $-(\sin x \cot x + \cos x \csc^2 x)$

5. $2\sec x \tan x + 3\sec^2 x + 3$

6. $2\sin x + 2x\cos x - 2x$

7. $[(x^2-1)\sec^2 x - 2x\tan x]/(x^2-1)^2$

8. $\sec x \tan x\,(1-\cos x) + (1+\sec x)\sin x$

9. $2\tan x \sec^2 x$

10. $2\sec^2 x \tan x$

14. $f(x) = \sin x$; $f(x) = \cos x$

15. $f''(x) = -f(x)$.

16. The derivative of $\sin x$ is $\cos x$. When $x = 0$, this is $\cos(0) = 1$. Thus, the tangent to the graph of $\sin x$ at the point (0, 0) has slope 1, which means it slopes upward at 45°.

12. The Chain Rule

17. $4(2x+1)$ **18.** $6(3x-1)$ **19.** $-(x-1)^{-2}$

20. $-4(2x-1)^{-3}$ **21.** $2(2-x)^{-3}$ **22.** $(1-x)^{-2}$

23. $(2x+1)^{-0.5}$ **24.** $-1.5(-x+2)^{0.5}$ **25.** $-4(4x-1)^{-2}$

26. $-2(x+7)^{-3}$ **27.** $-3/(3x-1)^2$ **28.** $-2/(x+1)^3$

29. $3(x^3-x)^2(3x^2-1)$ **30.** $-4x(2x^2-2)^{-2}$

31. $-5(2x-3)(x^2-3x-1)^{-6}$

32. $-6x/(x^2+1)^4$

33. $4(2s-0.5s^{-0.5})(s^2-s^{0.5})^3$

34. $-\dfrac{x}{\sqrt{1-x^2}}$

35. $-[(x+1)(x^2-1)]^{-3/2}(3x-1)(x+1)$

36. $6.2(3.1x-2) + 6.2/(3.1x-2)^3$

37. $2[(6.4x-1)^2 + (5.4x-2)^3][12.8(6.4x-1) + 16.2(5.4x-2)^2]$

38. $-2(x^2-3x)^{-3}(2x-3)(1-x^2)^{0.5} - x(x^2-3x)^{-2}(1-x^2)^{-0.5}$

39. $-56(x+2)/(3x-1)^3$ **40.** $3z^2(1-z^2)/(1+z^2)^4$

41. $3[(1+2x)^4 - (1-x)^2]^2[8(1+2x)^3 + 2(1-x)]$

42. $\pi\cos\left[\dfrac{\pi}{5}(x-4)\right]$ **43.** $-(2x-1)\sin(x^2-x)$

44. $(2.2x^{1.2} + 1.2)\sec(x^{2.2}+1.2x-1)\tan(x^{2.2}+1.2x-1)$

45. $\sec x \tan x \tan(x^2-1) + 2x\sec x \sec^2(x^2-1)$

46. $-\dfrac{\left(\dfrac{1}{\sqrt{2x+1}} - 2x\right)}{\left(\sqrt{2x+1} - x^2\right)^2}$ **47.** $\dfrac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

48. $\left(1-\frac{1}{x^2}\right)\sec^2\left(x+\frac{1}{x}\right)$

49. $-\sin(x)\cos(\cos(x))$

50. $2+3\left[2x+(2x+1)^3\right]^2(2+6(2x+1)^2)$

51. $(0.5x^{-0.5}+1.5x^{0.5})\ dx/dt$

52. $(1-r^{-2})\ dr/dt$

53. $8\pi r\ dr/dt$

54. $-4/27$

55. $e^{-2x}[-2\sin(3\pi x)+3\pi\cos(3\pi x)]$

56. $e^{5x}[5\sin(-4\pi x)-4\pi\cos(-4\pi x)]$

57. $1.5[\sin(3x)]^{-0.5}\cos(3x)$

58. $-\frac{x^2-2x}{(x-1)^2}\sin\left(\frac{x^2}{x-1}\right)$

59. $\frac{x^4-3x^2}{(x^2-1)^2}\sec\left(\frac{x^3}{x^2-1}\right)\tan\left(\frac{x^3}{x^2-1}\right)$

60. $2\frac{(2+e^x)\tan x\sec^2 x-e^x\tan^2 x}{(2+e^x)^3}$

61. $N'(6)\approx -32.12$ On January 1, 2003, the number of sunspots was decreasing at a rate of 32.12 sunspots per year.

62. **(a)** (**III**) **(b)** Increasing at a rate of 0.157 degrees per thousand years

13. Implicit Differentiation

1. $-2/3$

2. $4/5$

3. x

4. $-2x/3$

5. $(y-2)/(3-x)$

6. $(1-y)/(1+x)$

7. $-y$

8. $e^x y/(1-e^x)$

9. $-\frac{y}{x(1+\ln x)}$

10. $\frac{y(1+xy)}{x-\ln x}$ or $\frac{1+xy}{x(2-x)}$

11. $-x/y$

12. $-2xy/(x^2-2y)$

13.. $-(6+9x^2y)/(9x^3-x^2)$

14..$3y/x$

15..$(p+10p^2q)/(2p-q-10pq^2)$

16. $1/\sec^2 y$

17. $-1/\sqrt{1-x^2}$

18. $-\frac{y(1+\cos(xy))}{x(1+\cos(xy))}=-\frac{y}{x}$

19. $-[1+y\cos(xy)]/[1+x\cos(xy)]$

20. $1-y-\cos y)/(x-x\sin y)$

21. 1

22. -2

23. -0.03314

14. Related Rates

1. **(a)** $6/(100\pi)\approx 0.019$ km/sec **(b)** $6/(8\sqrt{\pi})\approx$ 0.4231 km/sec

2. 7.5 ft/sec

3. Monthly sales will drop at a rate of 26 T-shirts per month.

4. The y-coordinate is decreasing at a rate of 1 unit per second.

5. $\frac{130}{\sqrt{5}}\approx 58$ mph

6. $\frac{750}{12\pi}\left(\frac{\pi}{375}\right)^{2/3}\approx 0.82$ cm/sec

7. $\frac{\sqrt{1+128\pi}}{4\pi}\approx 1.6$ cm/sec

8. $600\pi\approx 1{,}885\ \text{cm}^3/\text{sec}$

9. Increasing at a rate of 1/40 radians/sec

10. 1/110 radians/sec

11. Decreasing at a rate of 1/2 m/sec

12. 1.5 m/s

13. $20/9\pi$ cm/sec

14. The section is called "related rates" because the goal is to compute the rate of change of a quantity based on a knowledge of the rate of change of a related quantity

15. Answers may vary: A rectangular solid has dimensions 2 cm $\times$5 cm $\times$10 cm, and each side is expanding at a rate of 3 cm/second. How fast is the volume increasing?

16. One would take the derivative of both sides with respect to time *t* and then substitute the known rates of change to solve for the unknown rate.

17. Linear

18. No. If one takes derivative with respect to time *t* of an algebraic equation relating two varying quantities x and y, then one obtains an algebraic equation in which dx/dt and dy/dt occur to the power 1

19. Let x = my grades and y = your grades. If $dx/dt = 2\ dy/dt$, then $dy/dt = (1/2)\ dx/dt$

20. If $y = mx + b$, then $dy/dt = m \cdot dx/dt$.

15. Differentials

1. $80\pi(r - 5)$

2. $\frac{2 - x}{\sqrt{3}}$

3. $A(x) = \sin a + \cos a\ (x - a)$; $\sin 3^{\circ} \approx 1/120 = 0.008\overline{3}$

4. $A(x) = 1 + 3(x-1) = 3x - 2$; 1.06

5. $\sqrt[3]{990} \approx 9.96667$ and $\sqrt[3]{1{,}010} \approx 10.03333$

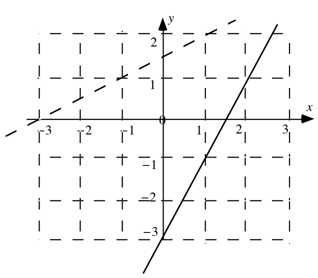
6. $\Delta A \approx 2\ \text{cm}^2$

7. $\Delta V \approx 60\ \text{cm}^3$

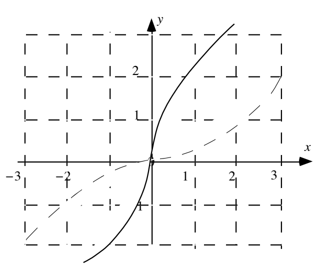
8. $0.236\ \text{cm}^3$

16. Logs and Exponentials

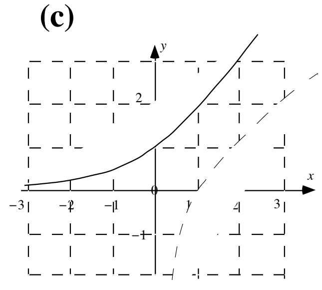
1. **(a)**



(b)



(c)

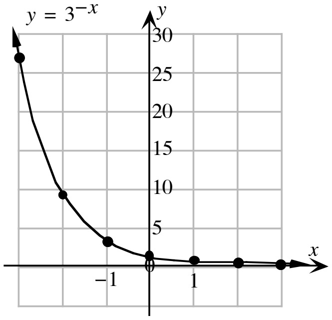


2. (a) $g(x) = \frac{x+1}{5}$ **(b)** $g(x) = (x-1)^{1/3} + 2$ **(c)** $k(x) = \frac{x}{x-1}$, with domain $(-\infty, 0]$

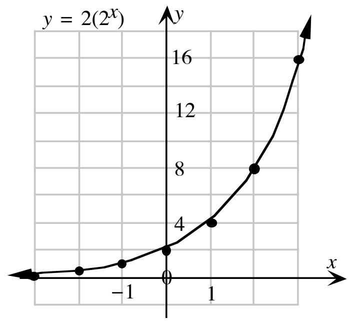
(d) $k(x) = \frac{x}{1-x}$ **(e)** $f(x) = ((x+1)/2)^{1/2}$, with domain $[-1,+\infty)$

(f) $g(x) = \frac{x}{1 - x}$, with domain $[0,1)$ **(g)** $g(x) = \frac{1}{x - 1}$, with domain $(1,+\infty)$

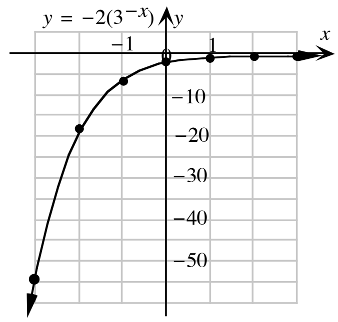
3. **(a)**



(b)



(c)



4. **(a)** $y = 2.1213\,(1.4142^x)$ **(b)** $y = 1.3867(1.4422^x)$

5. **(a)**

year	1950	2000	2050	2100
$C(t)$ parts per million	561	669	799	953

(b) 2020 ($t = 270$)

6.

Exponential form	$10^4 = 10{,}000$	$4^2 = 16$	$3^3 = 27$	$5^1 = 5$	$7^0 = 1$	$4^{-2} = \frac{1}{16}$
Logarithmic form	$\log_{10}10{,}000 =$	$\log_4 16 =$	$\log_3 27 =$	$\log_5 5 =$	$\log_7 1 =$	$\log_4 \frac{1}{16} = -2$

Exponential form	$(0.5)^2 = 0.25$	$5^0 = 1$	$10^{-1} = 0.1$	$4^3 = 64$	$2^8 = 256$	$2^{-2} = \frac{1}{4}$
Logarithmic form	$\log_{0.5}0.25 = 2$	$\log_5 1 = 0$	$\log_{10}0.1 = -1$	$\log_4 64 = 3$	$\log_2 256 = 8$	$\log_2 \frac{1}{4} = -2$

7. 1.4650
8. −1.1460
9. 0.2994
10. −0.7324
11. 1.3548
12. $1/(x-1)$
13. $1/(x\ln 2)$
14. $2x/(x^2+3)$
15. e^{x+3}
16. $-e^{-x}$
17. $4^x \ln 4$
18. $2^{x^2-1}\, 2x\ln 2$
19. $1 + \ln x$
20. $2x\ln x + (x^2+1)/x$
21. $10x(x^2+1)^4\ln x + (x^2+1)^5/x$
22. $3/(3x-1)$
23. $4x/(2x^2+1)$
24. $(2x-0.63x^{-0.7})/(x^2-2.1x^{0.3})$
25. $-2/(-2x+1) + 1/(x+1)$
26. $3/(3x+1) - 4/(4x-2)$
27. $1/(x+1) + 1/(x-3) - 2/(2x+9)$
28. $2/(x+1) - 9/(3x-4) - 1/(x-9)$
29. $\frac{4\ln(x^2)}{x} = \frac{8\ln x}{x}$
30. $1/(x+1) + 3e^x(x^3 + 3x^2)$
31. $e^x(\ln|x| + 1/x)$
32. $(2x-1)e^{x^2-x+1}$
33. $2xe^{2x-1}(1+x)$
34. $4(e^{2x-1})^2$
35. $-4/(e^x-e^{-x})^2$
36. $2(x-1)$
37. $\frac{1}{x\ln x}$
38. $\frac{1}{x|\ln x|\ln|\ln x|}$
39. $\frac{1}{2x\ln x\sqrt{\ln(\ln x)}}$
40. $y = (e/\ln 2)(x-1) \approx 3.92(x-1)$

41. $y = x$

42. $y = -\frac{4\ln 2}{3}(x-1) + 2$

17. Maxima & Minima

1. **(a)**. Absolute min.: $(-3, -1)$, relative max: $(-1, 1)$, relative min: $(1, 0)$, absolute max: $(3, 2)$

(b) Absolute min: $(3, -1)$ and $(-3, -1)$, absolute max: $(1, 2)$

(c) Relative min: $(-1, 1)$

(d) Relative min: $(-3, 0)$, relative max: $(-2, 1)$, absolute min: $(-1, -1)$, stationary non-extreme point: $(1, 1)$.

2. **(a)** Stationary minimum at $x = -1$

(b) Stationary minima at $x = -2$ and $x = 2$, stationary maximum at $x = 0$

(c) Singular minimum at $x = 0$, stationary non-extreme point at $x = 1$

(d) Stationary minimum at $x = -2$, singular non-extreme point at $x = -1$, singular non-extreme point at $x = 1$, stationary maximum at $x = 2$

3. Absolute max: $(0, 1)$, absolute min: $(2, -3)$, relative max: $(3, -2)$

4. Absolute min: $(-4, -16)$, absolute max: $(-2, 16)$, absolute min: $(2, -16)$, absolute max: $(4, 16)$

5. Absolute min: $(-2, -10)$, absolute max: $(2, 10)$

6. Relative max: $(-1, 5)$, absolute min: $(3, -27)$

7. Relative max: $(-1, 5)$, absolute min: $(1/2, -1/16)$

8. Relative min: $(-2, 5/3)$, relative max: $(0, -1)$, relative min: $(2, 5/3)$

9. Relative max: $(0, 0)$; absolute min: $(1/3, -2\sqrt{3}/9)$

10. Relative max: (0,□0), absolute min: $(1, -3)$

11. No relative extrema

12. Relative max: $(-3, -9/2)$, relative min: $(3, 9/2)$

13. Absolute min: $(1, 1)$

14. Absolute min: $(2, 2-\ln 4)$

15. Relative max: $(-1, 1+1/e)$, absolute min: $(0, 1)$, absolute max: $(1, e-1)$

16. Absolute max: $(0, 1)$

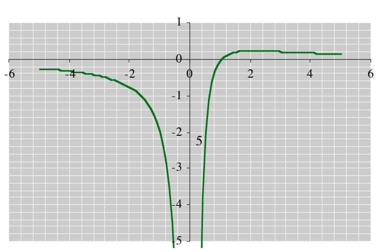
17. Relative max: $(-6, -24)$, relative min: $(-2, -8)$

18. Absolute max $(1/\sqrt{2}, \sqrt{e/2})$, absolute min : $(-1/\sqrt{2}, -\sqrt{e/2})$

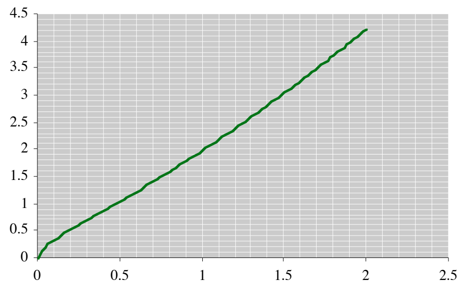
19. Absolute min: $(\frac{1}{e}, -\frac{1}{e})$

20. Absolute max of 1 at $x = 0, \pi, 2\pi, 3\pi$, absolute min of 0 at $x = \pi/2, 3\pi/2, 5\pi/2$

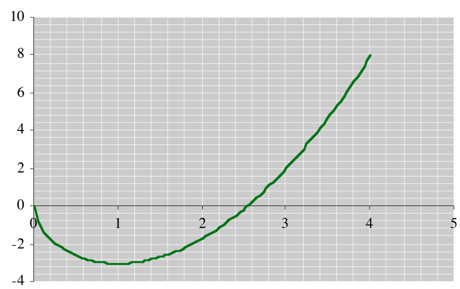
21.□Absolute maximum at $x = 2$



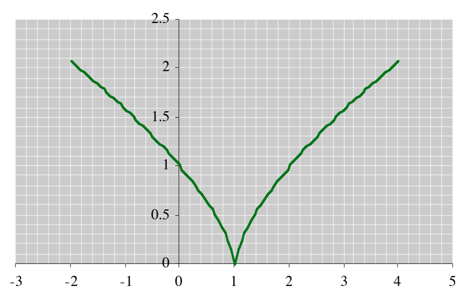
22. Absolute minimum at $x = 0$:



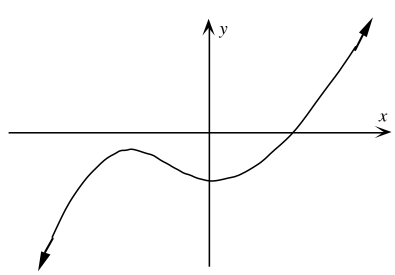
23. Relative maximum at (0, 0); absolute minimum at (1, −3)



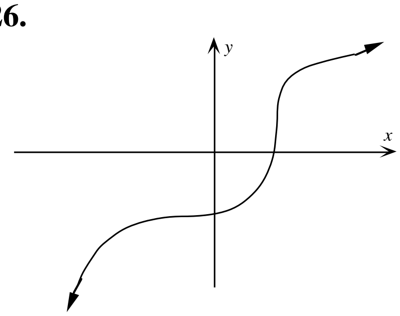
24. Absolute (singular) minimum at (1, 0)



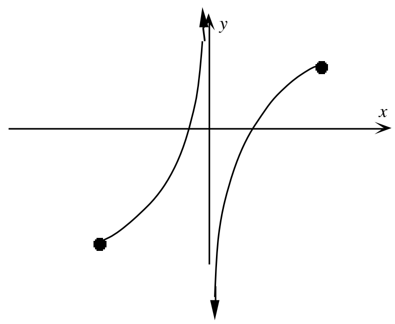
25.



26.



27.



28. Not necessarily; it could be neither a relative maximum nor a relative minimum, as in the graph of $y = x^3$ at the origin.

29. Not necessarily, both could be relative maxima, as in the case of $y = x^2;\ -1 \le x \le 1$

18. Applications of Maxima & Minima

1. $5 \times 10 = 50$ square feet

2. $p = \$10$

3. $l = 20$ in, $h = 10$ in, $w = 15$ in, $V = 3000$ in^3

4. (i) \$1.41 per pound **(ii)** 5,000 lbs **(iii)** \$7,071

6. 1.6 years, or year 2001.6; R_{max} = \$28,241 million

7. $x = \pm 1/\sqrt{3}$ for an area of $4/[3\sqrt{3}]$

8. $x = \pm 1/\sqrt{2}$ for a maximum area of 1

9. $t = 2.5$ or midway through 1972.; $D(2.5)/S(2.5) \approx 4.09$ The number of new (approved) drugs per \$1 billion dollars of spending on research and development reached a high of around 4 approved drugs per \$1 billion midway through 1972.

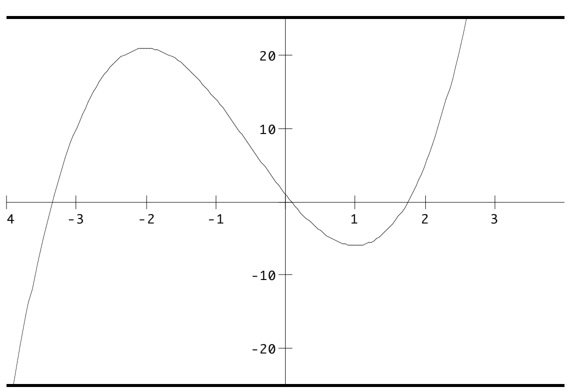
10. 15in $\times$30in $\times$30in

11. $\frac{65}{3}$ in.$\times$ $\frac{65}{3}$ in.$\times$ $\frac{130}{3}$ in.

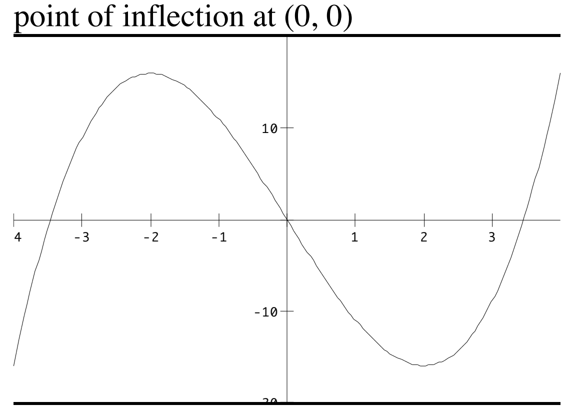
12. 40 laborers and 250 robots

19. Curve Sketching

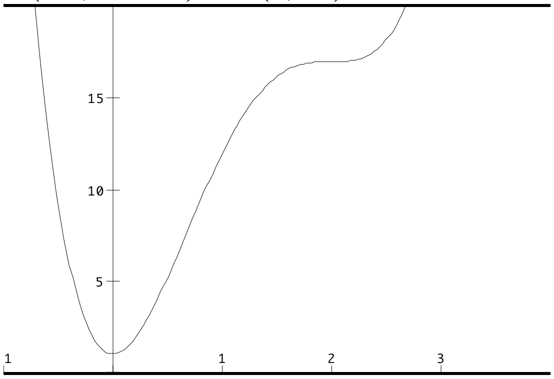
1. Relative max at $(-2,\ 21)$, relative min at $(1, -6)$, point of inflection at $(-1/2,\ 15/2)$



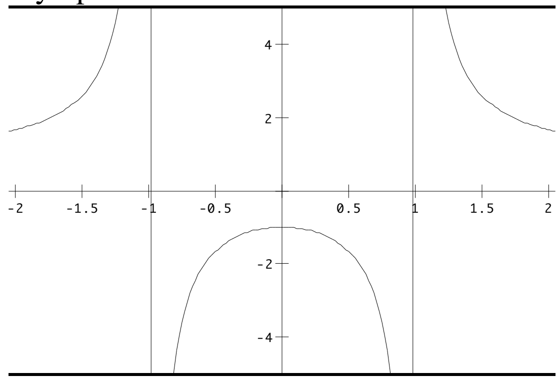
2. Absolute min at $(-4,\ -16)$ and $(2,\ -16)$, absolute max at $(-2,\ 16)$ and $(4,\ 16)$, point of inflection at $(0, 0)$



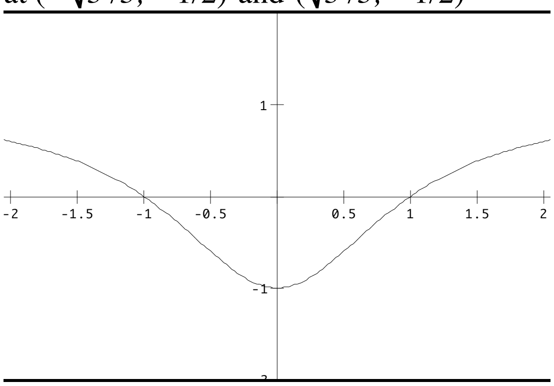
3. Absolute min at (0, 1), points of inflection at (2/3, 203/27) and (2, 17)



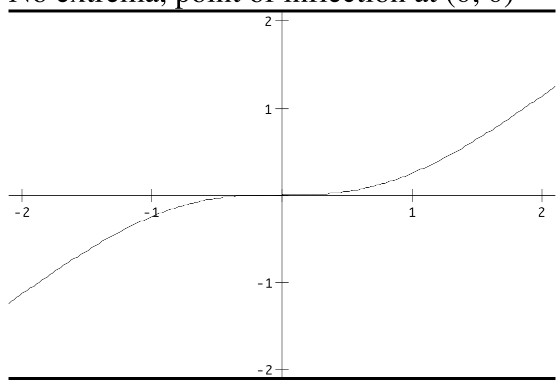
4. Relative min at (−2, 5/3) and (2, 5/3), relative max at (0, −1), vertical asymptotes: $x = \pm 1$



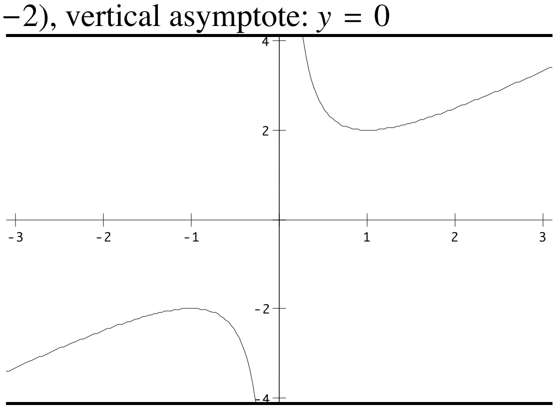
5. Absolute max at (−2, 3/5) and (2, 3/5), absolute min at (0, −1), points of inflection at $(-\sqrt{3}/3, -1/2)$ and $(\sqrt{3}/3, -1/2)$



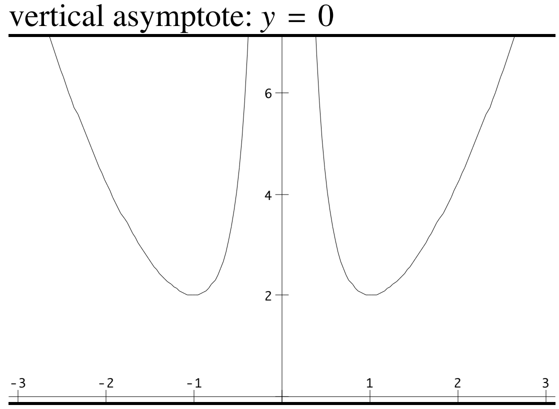
6. No extrema, point of inflection at (0, 0)



7. Relative min at (1, 2), relative max at (−1, −2), vertical asymptote: $y = 0$

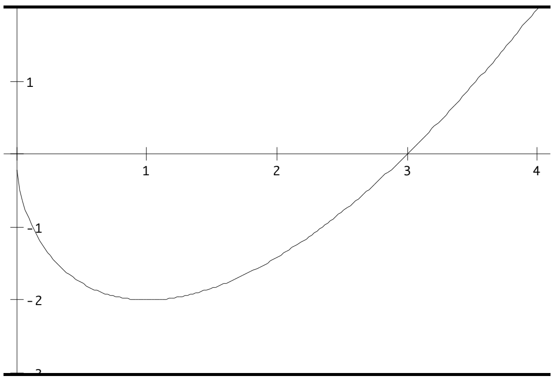


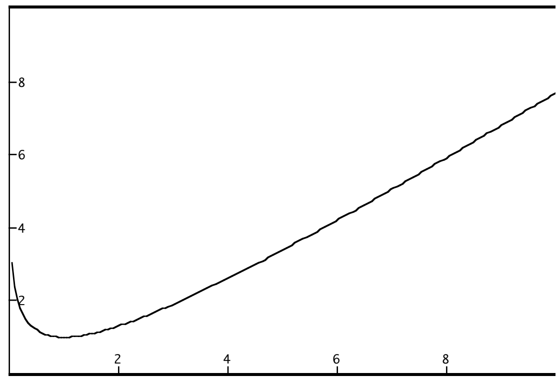
8. Absolute min at (−1, 2) and (1, 2), vertical asymptote: $y = 0$



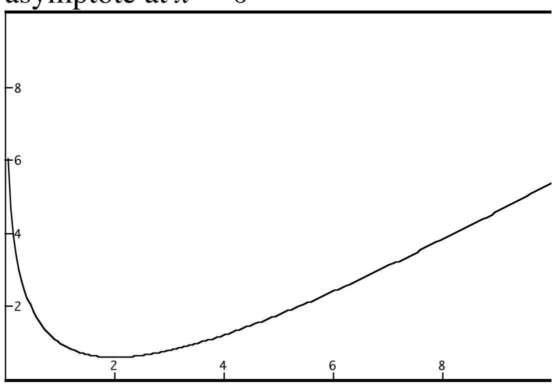
9. Relative max at (0,0), absolute min at (1,−2)

10. Absolute min at (1, 1), vertical asymptote at $x = 0$

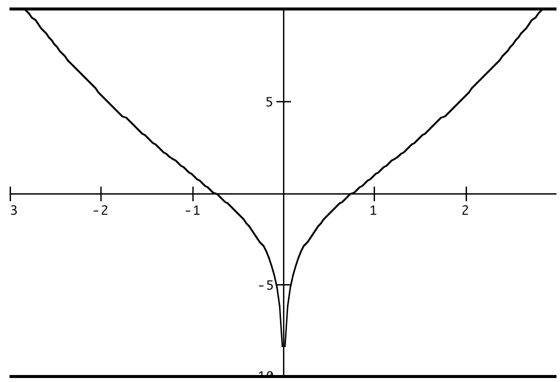




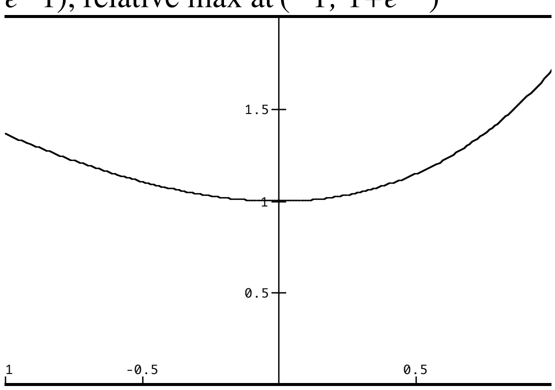
11. Absolute min at $(2,\ 2-\ln 4)$, vertical asymptote at $x = 0$



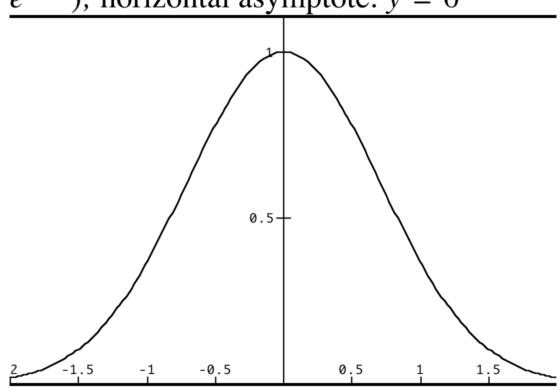
12. No relative extrema, point of inflection at $(1,1)$ and $(-1,\ 1)$, vertical asymptote at $x = 0$.



13. Absolute min at $(0,\ 1)$, absolute max at $(1,\ e-1)$, relative max at $(-1,\ 1+e^{-1})$



14. Absolute maximum at $(0,\ 1)$, points of inflection at $(-1/\sqrt{2}\ ,\ e^{-1/2})$ and $(1/\sqrt{2}\ ,\ e^{-1/2})$, horizontal asymptote: $y = 0$



20. Antiderivatives

1. $x^6/6 + C$
2. $x^8/8 + C$
3. $6x + C$
4. $-5x + C$
5. $x^2/2 + C$
6. $-\ 2\cos x + C$
7. $x^3/3 - x^2/2 + C$
8. $x^2/2 + \sin x + C$

9. $x + x^2/2 + C$
10. $\tan x + \sin x + C$
11. $-x^{-4}/4 + C$
12. $x^{3.3}/3.3 - x^{-0.3}/0.3 + C$
13. $u^3/3 - \ln|u| + C$
14. $\dfrac{2x^{3/2}}{3} + C$
15. $\dfrac{3x^{4/3}}{4} + C$
16. $3x^5/5 + 2x^{-1} - x^{-4}/4 + 4x + C$
17. $\ln|x| - \dfrac{2}{x} + \dfrac{1}{2x^2} + C$
18. $\dfrac{x^{0.9}}{0.3} + \dfrac{40}{x^{0.1}} + C$
19. $2e^x + 5\ln|x| + x/4 + C$
20. $12.2x^{0.5} + x^{1.5}/9 - e^x + C$
21. $-\cos x - 2\sin x + C$
22. $\sin x + \cos x + C$
23. $2\sin x + 4.3\cos x - 9.33x + C$
24. $-4.1\cos x + \sin x - 9.22\ln|x| + C$
25. $3.4\tan x + (\sin x)/1.3 - 3.2e^x + C$
26. $(3/2)\tan x - 1.3\cos x - e^x/3.2 + C$
27. $\dfrac{2^x}{\ln 2} - \dfrac{3^x}{\ln 3} + C$
28. $\dfrac{100(1.1^x)}{\ln(1.1)} + C$
29. $x^2/2 - 2\ln|x| + C$
30. $f(x) = x^2/2 + 1$
31. $f(x) = 2e^x + x - 2(e+1)$
32. 49/3
33.(b) 25/16 ft.

21. Change of Variables

1. $(3x+1)^6/18 + C$
2. $(-2x+2)^{-1}/2 + C$
3. $-e^{-x} + C$
4. $1.6(3x-4)^{3/2} + C$
5. $2e^{(0.6x+2)} + C$
6. $(3x^2+3)^4/24 + C$
7. $(x^2+1)^{2.3}/4.6 + C$
8. $x + 3e^{3.1x-2} + C$
9. $(7.6/3)\sin(3x-4) + C$
10. $-(1/6)\cos(3x^2-4) + C$
11. $-2\cos(x^2+x) + C$
12. $(e^x - e^{-x})/2 + C$
13. $2(3x^2-1)^{3/2}/9 + C$
14. $-(1/2)e^{-x^2+1} + C$
15. $-(1/2)e^{-(x^2+2x)} + C$
16. $(1/6)\tan(3x^2+2x^3) + C$
17. $-(1/6)\ln|\cos(2x^3)| + C$
18. $3\ln|\sec(2x-4) + \tan(2x-4)| + C$
19. $(x^2+x+1)^{-2}/2 + C$
20. $(2x^3+x^6-5)^{1/2}/3 + C$
21. $(x-2)^7/7 + (x-2)^6/3 + C$
22. $4[(x+1)^{5/2}/5 - (x+1)^{3/2}/3] + C$
23. $3e^{-1/x} + C$
24. $\cos(1/x) + C$
25. $\ln(e^x + e^{-x}) + C$
26. $(-e^{-x^2+1} + e^{2x})/2 + C$

22. Definite Integral & Fundamental Theorem of Calculus

1. 0 **2.** 0 **3.** 2π **4.** 2π **5.** $A(x) = x^3/3$ **6.** $A(x) = \ln x$
7. $A(x) = x^2$ if $0 \le x \le 2$; $4 + (x-2)$ if $x > 2$
8. 14/3
9. 5

10. 0

11. 40/3

12. ln(2)

13. 0

14. 1

15. 1

16. 1

17. $3(1-e^{-1})$

18. 2/3

19. 0

20. 4^6-1

21. 1

22. $(e^1 - e^{-3})/2$

23. $e - 1/e$

24. $e^{2.1}-e^{-0.1}$

25. $(5^{3/2}-1)/6$

26. 0

27. (3/2) ln(3)

28. (1/3)[ln26 − ln7]

29. (1/4)[ln13 − ln3]

30. $\dfrac{0.1}{2.2\ln(1.1)}$

31. $\dfrac{1.1}{3\ln(2.1)}$

32. $e - e^{1/2}$

33. $(2/3)(\ln 2)^{3/2}$

34. 2 − ln 3

35. 4(1 − ln 3)

36. −4/21

37. −15/14

38. $3^{5/2}/10 - 3^{3/2}/6 + 1/15$

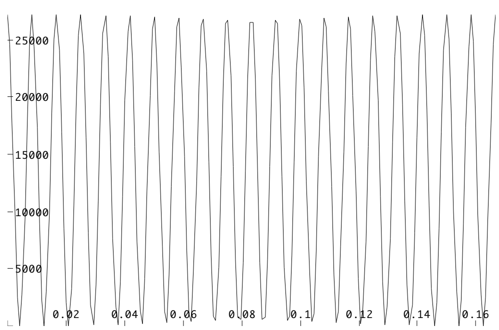
39. −8/15

40. 1/2 **41.** 16/3 **42.** 56/3 **43.** $[1-e^{-1}]/2$ **44.** 79 sunspots

45. $P(t) = 7.5\sin[(\pi/26(t-13)] + 12.5$; 7.7%\

46. **(a)** Average voltage over [0, 1/6] is zero; 60 cycles per second.

(b)



<0 ≤ y ≤ 28000; peaks at height of 27,225>

(c) 116.673 volts.

48. $F(x) = \begin{cases} 0 & \text{if } x < 0; \\ x & \text{if } x \geq 0. \end{cases}$ $F'(x) = f(x)$ for every $x \neq 0$ (since $F(x)$ is not differentiable at $x = 0$).

49. **(a)** $A'(x) = 1/x$ **(b)** $A(1) = 0$

50. (b) (i) e^{-x^2} **(ii)** $\frac{\sqrt{\pi}}{2}\operatorname{erf}(x)$ **(iii)** $-\frac{4x}{\sqrt{\pi}}e^{-(x^2-1)^2}$

51.. $\mathcal{J}(x) = \int_1^x \frac{\sin t}{t}\,dt$

23. Area Between Curves

1. 8/3
2. 2
3. 1
4. $e - 3/2$
5. 2/3
6. 10,235/66
7. 3/10
8. 1/20
9. 4/15
10. $2\ln 2 - 1$
11. 8ln4 + 2e - 16
12..1/2
13. $3\sqrt{3} - 2$

24. Continuity and the Intermediate Value Theorem

1. Answers may vary: $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0. \end{cases}$ has $f(-1) = -1$, $f(1) = 1$, but $f(x)$ is never 0.

2. Let $A(x)$ = area of a square with side x. Then $A(r) = r^2$ and $A(2r) = 4r^2$. Since $A(r) = r^2 < \pi r^2 < 4r^2 = A(2r)$, the result follows from the IVT.

5. Let $f(t)$ = distance along the road from the bottom on the way up as a function of t, and let $g(t)$ = distance along the road from the bottom on the way down. Take $t = 0$ to be 12:00 noon. Then $h(t) = f(t) - g(t)$ has $h(0) = -h(12)$. Therefore, there exists at leat one t with $h(t) = 0$.

25. The Mean Value Theorem (MVT)

1. $\pm 2/\sqrt{3}$

27. Computations of Volumes, Length, Area

1. Calculate the volumes of the following solids of rotation:

(a) 625π **(b)** $\frac{25\pi}{2}$ **(c)** $\frac{2{,}304\pi}{5}$

(e) . $\frac{4}{3}\pi ab^2$ **(f)** . $\frac{4}{3}\pi a^2 b$ **(g)**. $\frac{4}{3}\pi ab^2$

2. $\frac{8\pi}{3}$
3..2π
4.. $2\sqrt{2} - 1$

5. **(a)** $2\sqrt{2} - 1$ **(b)** $\frac{2}{3}(2\sqrt{2} - 1)$ **(c)** $\frac{8}{15}(\sqrt{2} + 1)$