Supplement C-The Silver Dollar Game

Overview

The Silver Dollar Game consists of putting a number of coins on a grid. Two players alternate turns. You may move one coin any number of squares to the left, but you may not jump on, or over, any other coin. The person who makes the last valid move wins.

We will illustrate with a grid that has 20 squares labeled 0 – 19. In figure 1, there are coins on squares 2, 7, and 11. If it's your turn, you may move one and only one of the three coins. The coin on square 2 may be moved to square 1 or square 0. The coin on square 7 may be moved to squares 6, 5, 4, or 3. The coin on square 11 may be moved to 10, 9, or 8.

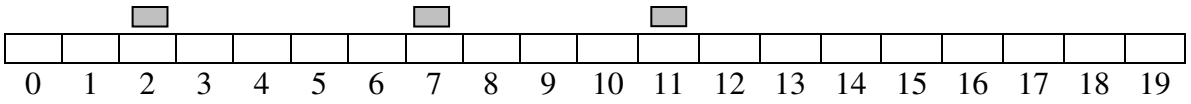

Figure 1

The game is over when the coins are on squares 0, 1, and 2. (See figure 2.) The person (or computer!) who moves the rightmost coin to square 2 is the winner.

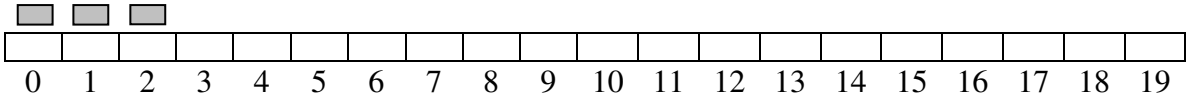

Figure 2

Winning Strategy

The strategy is intimately related to the strategy for the Game of Nim which by now you should be familiar. The notes assume you know how to solve a Nim problem.

Begin by grouping the coins in pairs from right to left. We will begin with an example containing an even number of coins. Consider the example in figure 3 with 6 coins.

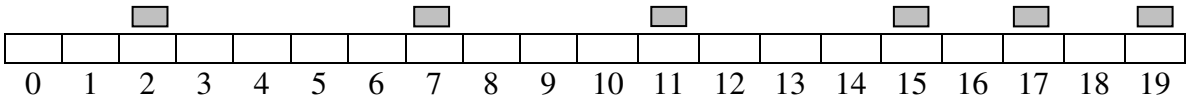

Figure 3

The paired coins are those on squares 17 and 19, 11 and 15, and 2 and 7.

You are in a winning position if you ultimately leave your opponent with no squares between the pairs as in figure 4. Why?

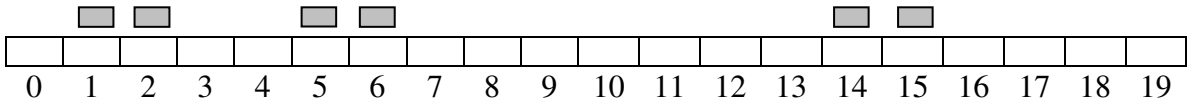

Figure 4

How do you get to such a situation? Notice that the number of squares between the pairs in figure 3 (from right to left) is 1, 3, and 4 respectively. You want these numbers eventually to be zero.

See the connection to Nim?

It's as if you had 1, 3, and 4 matches in piles 1, 2, and 3 respectively. In Nim, you win if you pick up the last match. Said in another way, you win in Nim if you leave the opponent with zero matches in all piles. You win in this game if you leave the opponent with zero spaces between the pairs. Get it?

OK, but what if there are an odd number of coins?

Suppose there are seven coins. Do you agree that you are in a winning position if you ultimately leave your opponent with the leftmost coin on square 0, and with the others paired and having no spaces between them as in figure 5?

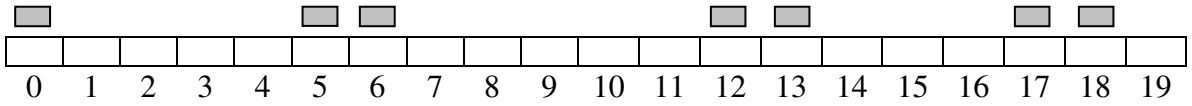

Figure 5

How can you assure this?

Suppose figure 6 gives the initial configuration of the seven coins.

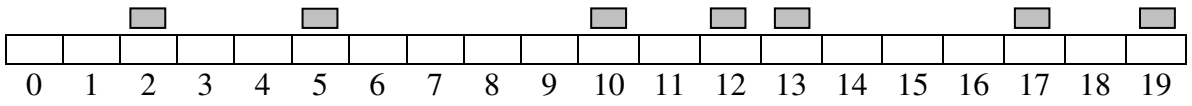

Figure 6

As instructed earlier, pair the coins from right to left. The leftmost coin is not paired, but as in figure 5, you want it to be on square 0. Converting the game of figure 6 to Nim language, you have 4 piles of matches with 1, 0, 4, and 2 matches respectively. Note that in this case, the number of matches in the last pile is the number of the square on which the leftmost coin lies. (Be sure to number the leftmost square 0 and not 1!) Once again you win if you leave the opponent with zero matches in all piles. This is precisely the situation in figure 5.

Example:

Solve the game of figure 6.

This problem can be reworded as follows: Suppose there are coins on squares 2, 5, 10, 12, 13, 17, 19. Determine the strategy. If you should go first, indicate your move (i.e. state which coin you are moving and the square to which you will move the coin.) If you should go last, there is nothing else to say.

The solution to the Nim Game corresponding to figure 6 is to take one match from pile 3. This means you want the number of spaces between the paired coins on squares 5 and 10 to decrease by one. The only way to do this is to move the coin on square 10 to square 9. (Remember, you must move the coins to the left.) This is your first move.

Two final points:

There can be any number of squares on a grid. To be consistent, you should always number the first square 0.

Based on the rules of Nim, it is possible that your strategy for your first move may be to go last. Verify that this is the case if the initial configuration contains coins on squares 4, 7, 13, 15, 24, and 28.

Exercises:

1. Suppose coins are on the squares given below. Convert the Silver Dollar Game problem to a Nim problem (i.e. give the piles and the number of matches in each pile).

a) 2, 6, 11, 17
b) 0, 7, 16
c) 8, 10, 15, 23, 39

In exercises 2-9 you are given the squares on which there are coins. Determine the strategy for each of the following Silver Dollar Games. If you should go first, indicate your move (i.e. state which coin you are moving and the square to which you will move the coin.)

2. 0, 7, 16
3. 4, 19, 25
4. 2, 18, 22, 35
5. 3, 7, 13, 14
6. 8, 13, 15, 23, 39
7. 3, 9, 16, 17, 22
8. 8, 11, 16, 27, 31, 40
9. 12, 14, 25, 27, 37, 40, 47

Answers:

1. a) Pile 1: 6 matches
   Pile 2: 3 matches

   b) Pile 1: 8 matches
   Pile 2: 0 matches

   c) Pile 1: 15 matches
   Pile 2: 4 matches
   Pile 3: 8 matches

2. Move the coin on square 16 to square 8.
3. Move the coin on square 25 to square 24.
4. Move the coin on square 18 to square 15.
5. Move the coin on square 7 to square 4.
6. Move the coin on square 39 to square 33.
7. Move the coin on square 3 to square 2.
8. Go last.
9. Move the coin on square 37 to square 28, or move the coin on square 25 to square 18, or move the coin on square 12 to square 5.