

Supplement A - The 5 Card Trick

You select any 5 cards from the deck. I place one card face down on the table, and line up the other 4 cards so everyone can see them (on the ledge of the chalkboard, for example.) I input the 4 visible cards, and the computer identifies the card that is face down.

How do we do this?

Because a deck of cards has only 4 suits, any set of 5 cards must have at least 2 cards of the same suit (the Pidgeonhole Principle). You are to pick 2 such cards. You may have a lot of choices; if all 5 cards are of the same suit, for example, you can pick any 2 cards. When you have a choice, I suggest picking the 2 cards for which the computations described below are easiest, but it doesn't really matter. The hidden card will be one of these 2 cards. You will place the other card in the second position from the left in the 4-card lineup. **The suit of the second card is the suit of the hidden card.**

How do we determine which of these two cards is the hidden card?

Any 2 cards have a value no more than 6 apart (considering A=1, J=11, Q=12, K=13). For example, if the 2 cards are a 7 and a Q, the Q is 5 more than the 7. If the 2 cards are a 4 and a Q, the 4 is 5 more than the Q when we wrap around past K (Q, K, A, 2, 3, 4). Pick 2 cards of the same suit and **hide the one that is at most 6 more than the other. The other card will be the second card of the four that are shown to the computer.** If the cards are 7 and Q, hide the Q and show the 7 in the second position. If the cards are 4 and Q, hide the 4 and show the Q.

But how does the computer know the face value of the hidden card?

If you can designate a number between 1 and 6, the computer can add that number to the value of the 2nd card to get the value of the hidden card.

But how do you determine the number between 1 and 6?

There are 6 ways to arrange the other 3 cards that go in positions one, three and four. Label these cards 1, 2, and 3, where 1 is the lowest, 2 is the next lowest, and 3 is the highest. For example, if the other 3 cards are 8, 10, and K, the 6 permutations are:

1: 8 10 K i.e. 1, 2, 3 = +1

2: 8 K 10 i.e. 1, 3, 2 = +2

3: 10 8 K i.e. 2, 1, 3 = +3

4: 10 K 8 i.e. 2, 3, 1 = +4

5: K 8 10 i.e. 3, 1, 2 = +5

6: K 10 8 i.e. 3, 2, 1 = +6

For example, to indicate that you need to add 5 to the value of the 2nd card, and if the other 3 cards are 8, 10, and K, then the 1st, 3rd, and 4th cards should be K, 8, and 10.

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But what happens if 2 (or even 3) of the other cards have the same value?

In this case, use the order of the suits in bridge to settle the tie between any cards. From lowest to highest, these are: clubs, diamonds, hearts, spades.

Here are some examples (using C, D, H, S for club, diamond, heart, spade).

The 5 cards are: 2H, 3C, 10D, JS, KH.

The 2 and K have the same suit and the 2 is 2 more than the K (i.e. K, A, 2).

Hide 2H. Display 3C KH JS 10D. Note: 3 J 10 is in the order 1, 3, 2 = +2. Adding 2 to the 2nd card (the KH) gives 2H, the hidden card.

The 5 cards are: AH, 2H, 3H, 5H, 9H

There are many choices. Here is a selection that makes the calculations easy and one that makes them slightly harder.

Easy: Hide 2H. Display 3H AH 5H 9H.

Hard: Hide AH. Display 5H 9H 2H 3H.

The 5 cards are: KC, KD, KH, KS, 4C.

Hide 4C. Display KH KC KS KD.

Exercises:

Determine the hidden card for each of the following sequences of cards:

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|----|-----|-----|----|----|----|----|-----|----|----|
| 1. | 2H | 7D | 6C | 5H | 6. | AS | 10S | AC | 3H |
| 2. | 10C | 6C | 8H | 3S | 7. | AS | 10S | 3H | AC |
| 3. | JC | QD | 2H | 4H | 8. | 7D | JC | QC | 7S |
| 4. | 9S | KH | KD | QC | 9. | 4D | 4H | 4S | 4C |
| 5. | AC | 10S | AS | 3H | | | | | |

You are given the following 5 cards. Which card should you hide, and how should the remaining cards be arranged?

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|-----|----|----|-----|----|-----|
| 10. | 5S | 8S | 7D | 2H | 4C |
| 11. | 2H | 6C | QH | 3S | 10D |
| 12. | KS | 6C | QH | 9S | JD |
| 13. | 2C | 9C | 10C | AS | 3S |
| 14. | 5S | 4D | 5H | KH | QC |
| 15. | QD | QS | 8D | 8H | 8C |
| 16. | 9D | 9H | 9S | 9C | 2S |
| 17. | 7D | 7H | 7S | 7C | QD |