MATH 12.3 MATHEMATICAL EXCURSIONS Fall '03
MWF 10:10 - 11:05 AM ROOM 028 Breslin
Instructor: Dr. David Knee

Textbooks (both are paperbacks):
"Strength in Numbers" by Sherman Stein, Wiley
"The Code Book" by Simon Singh, Anchor Books

Home Work

Chap. 1 of Stein

1. React, explain, guess at meanings of items on list p.7-8. This is preview of this text's content.

Chap. 2

1. Find 2 other numbers that are the sum of 2 squares. (p. 11)
2. Find another perfect number. (p. 11)

2.5 Find 2 deficient and 2 abundant numbers.

3. Is 3/5 really between 1/2 and 2/3? (p. 11)
4. How can you tell if a/b is between c/d and e/f ?
5. Our natural numbers are written in the base 10. Explain. (p.13)
6. Count to twenty in base 8.
7. What's so interesting or famous about 3, 7, 40, & 666 ?

Chap. 3

1. Think of 2 things to measure about the members of our class, 1 that's straightforward and another that not. Give possible definitions and procedures to obtain the measures.

2.

	Last year	This year
Joe	95 lbs.	106 lbs.
Mary	111 lbs.	102 lbs.

Find the change (increase or decrease) and the % change in Joe's and Mary's weights.

Chap. 4

1. Find the mean & the median of : 2, 5, 7, 4, 4, 10, 3.
2. Estimate the mean & median of physicians' weekly income from the graph on p. 24.

Chap. 5

1. Understand the line graphs on p. 28-31 (murders, traffic accidents).
2. Criticize the author's conclusion of paragraph 2, p. 29.

Chap. 6

1. p. 33: What do you know about these Math topics?

2. How long a string can you make with no substring of the form x^2 from alphabet {a, b} ?

3. What do you know about Euclid's geometry of "The Elements"?

4. How would you construct 75^o and 37.5^o angles with ruler and compass alone?

5. Find 3 sets of whole numbers (natural numbers) for which $x^2 + y^2 + = z^2$ (Pythagorean Triples).

6. Find a simple map that needs 4 colors.

7. How many colors does the "Pizza Map" of n slices need ?

8. Evaluate $1/1^2 + 1/2^2 + 1/3^2 + \ldots + 1/6^2$.

9. Evaluate $1/1^3 + 1/2^3 + 1/3^3$.

10. Give 2 examples each of members of N, Z, Q, R, R-Q.

11. If a is in N then 3a, 4a, 5a form a Pythagorean Triple.

12. Solve $1 = g^2 - g$ to get the Golden Ratio.

13. Prove/argue that the sum of the angles of any triangle is 180^o.

Chap. 7

1. Look up M.C. Escher and see a picture of a staircase where going up is going down.

Chap. 8

1. Verify that the knots pictured on p. 51 are the same or different by constructing them with string.

2. Find the divisors (factors) of 20.

3. Find the multiples of 8.

4. Find the next 5 primes after 23.

5. Is 151 a prime?

6. Compute 3^6 .

7. If n is in N, then $n^3 - n$ is divisible by 6. (Hint: factor it).

8. Check Euler's Theorem for $a = 3$, $b = 5$ and $n = 4$.

9. Find the prime factorization of 1050.

10. Find the prime factorization of 1739.

11. Verify that 496 is perfect.

Chap. 9: None

Chap. 10

1. Construct a bar graph where the vertical axis represents % of the American Workforce and the horizontal axis has 6 intervals corresponding to the 6 levels of Math background needed for work. Label your graph, "Math that American Workers Need".

Chap. 11: None

Chap. 12

1. Count to twenty in base 4.
2. Add in base 4: 23 + 1032 . (These numbers are written in base 4)
3. Multiply in base 4: 23x1032 . (The numbers are written in base 4)
4. Draw the number line for R and label 10 points.
5. Draw the Cartesian Plane (R^2) and label 5 points.
6. Add in base 2: 10011 + 110 . (The numbers are written in binary)
7. Multiply in base 2: 11x1010 . (The numbers are written in base 2)
8. Count to twenty in binary.
9. Carry out the 2 experiments concerning the sum of the angles of a triangle in the text P. 89-90.
10. Prove sum of the angles of a triangle = 180^o.
11. Do you agree with the author on p. 92, 2nd & 3rd paragraphs?
12. Which is larger 5/6 or 1/2 ?
13. Add 2/5 + 1/10.
14. What are the formulas for perimeter and area of a rectangle and a circle ?

Chap. 13: None

SINGH, Chaps. 1-3.

Chap. 1

1. Compute 1!, 2!, ... , 7!
2. What's 20! ?
3. Check book's estimates on p. 7 for the 35-letter sentence. Ignore the spaces. a) # of arrangements, b) time for"brute force method".
4. How many recognizably different permutations (arrangements) are there of "iron", "mississippi"?
5. Decrypt this 2-line rail fenced message:
IMSCECDSATRPAEOAAERTOEMSESYRYU
6. ENCRYPT MESSAGES OF YOUR OWN THAT YOUR NEIGHBOR WILL DECRYPT IN CLASS (SEPARATE SHEETS OF PAPER) in:
   a) railfence 2-line.
   b) railfence 3-line.
   c) Caesar 3-shift.
   d) Caesar 4-shift.
   e) Substitution using phrase "mathematical excursions" (p.13).
   f) Atbash (p.26).
   g) Pigpen Cipher (appendix D).
7. Check author's estimates on p.12 for a) # of keys and b) time for brute force method.
8. Decrypt the pigpen message on p. 375.
9. Decrypt the message on p. 20 without looking ahead. If you need a hint, use only table 2 on p. 21.
10. Practice decryption of general substitution ciphers with

"Cryptoquotes" from Newsday.

11. See "Cipher Challenge", p. 351-66 and solve stage 1.

SINGH Chap. 2

1. Solve this Cryptoquote from Newsday:

ZXXSVMU RHJF BQHLPXY IXQQ SFXYYXN IVQQ ZXXS RHJ DFHT QHHZVMU

PCFN SFXYYXN. -- BHQXTCM BHG

2. Encrypt your own messages using these cipher systems (separate sheet of paper, your name on top):

a) Vigenere Cipher using MANINTHEIRONMASK as keyword,

b) the homophonic substitution cipher of Table 5, p. 53,

c) the Alberti Cipher of p. 45-6,

d) the Playfair Cipher of appendix E,

e) Beale Cipher #2, p. 92, and

f) a superencipherment (p. 97)-- pick any 2 ciphers.

3. See Morse Code, table 6 on p. 62. How many different 'strings' of dots and dashes are possible of length < or = 4 ?

4. Summerize the steps of Babbage's technique for decrypting a message written in the Vigenere Cipher.

5. Draw the frequency distribution bar graph for L_3 using only the first 8 lines of figure 13, p. 70. Use it to guess at the identity of L_3.

SINGH Chap. 3 to 2/3 p. 124)

1. Encrypt your own messages using these cipher systems:

a) the ADFGVX code of p. 378-80 (appendix F),

b) the Vigenere Cipher using an easy-to-memorize keyword or phrase that's as long as the message itself (p. 116-20), and

c) the one-time pad of figure 30, p. 121.

2. Describe a decryption method for a message written in

a) the ADFGVX Cipher,

b) ex. 1b) cipher.

3. Check the author's estimate for the # of possible keys for a message of 21 letters using the 1-time pad (p. 121).

STEIN book Part 2, problems

Chap. 14

1. (Re)do Chap. 6, prob. 10.
2. (Re)do Chap. 12, probs. 4 & 5.
3. Compute $\sqrt{81}$ and $\sqrt{112}$.

Chap. 15

1. What's bigger, a googol or 6^{210} ?

2. (Re)do Chap. 8, probs. 8 & 10.

3. Continue table on p.114 for 16 thru 20, and also calculate a 4th line for #odders - # eveners.

4. If 10,000 grains of sand fill up a cubic inch, how many would it take to fill the whole Earth?

5. Fill in these four tables:

+	even	odd
even		
odd		

x	even	odd
even		
odd		

+	evener	odder
evener		
odder		

x	evener	odder
evener		
odder		

<u>Chap. 16</u>

1. Collect data on picking 1 out of 2 at random; 1 out of 3.

2. Experiment with 4 cans (p. 123). Guess at the probability of winning if you always stick with or always switch your original choice.

3. You are presented with 5 closed doors; behind 1 is a new car but there's nothing behind the others. You choose 1 door, but before you open it, the MC opens 2 other empty ones. Should you switch your choice or stick with the original? What is the probability of winning the car if you switch? If

you stick?

4. Same question with n closed doors where the MC will choose r empty ones (2<n and 0<r<n-1).

Chap. 17

1. Calculate first without then with a calculator: $8^{2/3}$, $32^{-3/5}$.
2. Estimate $(5x10^{20})/(60x60x24)$, then calculate with a calculator.
3. Create a table (like that of p.134) for 3^n from n = -3 to 3.
4. A^b is sometimes written as a ^ b. Is ^ commutative, associative, distributive over +, distributive over * ?

Chap. 18

1. (Re)do Chap. 6, probs. 8 & 9.
2. Find .999999.... , .33333...., .12121212.....
3. Find 1 + (.4) + (.4) ^ 2 + (.4) ^ 3 +.....
4. Find 1 + (.4) + (.4) ^ 2 + (.4) ^ 3 + ... + (.4) ^ 19.
5. Find 1 + 1/2 + 1/3 + 1/4 + ... + 1/7.
6. Using the proof on p. 141, how far out in the Harmonic Series should you go to be sure that the sum now exceeds 1000?
7. Does the last sum on p. 142 (1/(1x2) + 1/(2x3) + 1/(3x4) + ...) converge or diverge ? Hint: See prob. 8 of Chap. 6 + Euler's result on p. 37 + the method in the proof on p. 141.
8. Notice that 1/(k*(k+1)) = 1/k - 1/(k+1). Use this to find the actual sum of the (telescoping) series in prob. 7.

Chap. 19

1. John Doe deposits $10,000 in a bank that must keep 10% of all deposits on reserve. The bank then lends the rest of that deposited $ to Ms. A which she promptly deposits into that same (or another similar) bank. And so on, ad infinitum. All these people believe they have (all together) how much $? What is the "multiplier"?
2. Same set up, but Mr. Doe deposits $D and the bank must keep t of its deposits on reserve, where 0<t<1.
3. Same set up with D = $10,000 again , t = .2, and the bank pays 5% interest on deposits but charges 8% interest on loans. Assume the infinitely many transactions occur instantly and that interest is computed simply. How much $ does the bank make at the end of a year?

Chap. 20

1. A = 7/12, b = 5/16. Find a+b, a-b, a*b and a/b.
2. Use 3 ways to determine which is larger, 6/17 or 7/19.

Chap. 21

1. Show that $\sqrt{3}$ is irrational. You may use the Lemma that if x is in N and x ^ 2 is divisible by 3 then so is x.

2. Redo prob. 1 for any prime p, not just 2 or 3 , since the corresponding Lemma continues to be true.

3. Find a 3 place decimal approximation to $\sqrt{5}$ by the method of "guess and refine" using your calculator to multiply only.

Chap. 22

1. Gather the data of the Table on p. 169 as the author suggests, to further verify the Pythagorean Theorem.

2. Find the proof of the Pythagorean Thm in figure 7 of p. 170.

3. Verify the 2 Babylonian assertions on bot. p.170 & top p. 171.

4. Draw a 6 or more sided convex (i.e. no indentations) polygon on a geoboard & find its area by subtraction.

5. Compute how far you can seeon the Earth's surface (to the horizon) given the 3 different eye heights on p. 172.

Chap. 23

1. Use inner & outer regular hexagons to get lower & upper bounds for π. (Bot. p.176)

2. Verify that the Archimedes bounds give the simpler bounds given on top p. 177.

3. Find Area & Circumference of a circle with radius 5.

4. Find the Surface Area and Volume of a ball of radius 4.

5. Show that the Circumference of a circle is irrational if the radius is rational. What if the radius is irrational?

6. Find the Surface Area & Volume of a right circular cylinder whose base radius is 3 and whose height is 7.

7. (p. 181) Show that the volume of a ball is a bit over 1/2 that of the circumscribing cube and exactly equal to 2/3 that of the circumscribing cylinder.

Chap. 24

Graph these equations:

1. $y = 2x^2$ 2. $y = 4x$ 3. $3x + 2y = 6$ 4. $x^2 + y^2 = 10$
5. $y = 2^x$ 6. $y = x^3$ 7. $y = 1/x$

Chap. 25

1. Compute $(-4)(2^{-3} - 5)^{-2}$

2. Law or Not?: a) $\sqrt{(a + b)} = \sqrt{a} + \sqrt{b}$ b) $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$
c) $a!b! = (ab)!$ d) $a!b! = (a + b)!$ e) $(a + b)/c = a/c + b/c$ f) $(-a)^b = -a^b$

Chap. 26

1. If A & B are disjoint and both co-numerous with N then so is AUB.

2. If A is finite and B is co-numerous with N then so is AUB.

3. Show that Z and Q are co-numerous with N.

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SINGH, 2nd part: rest of Chap. 3. Parts of Chaps. 4 & 5.

<u>Chap. 3, from p. 124</u>

1. Encrypt "face" using the Enigma-like device of fig. 33, p.129.
2. Encrypt "ace" using the Enigma-like device of fig. 34, p.129.
3. Encrypt "fed" using the Enigma-like device of fig. 35, p.131.
4. Encrypt "bed" using the Enigma-like device of fig. 36, p.132. Then decrypt the result (note that fig. 36 is similar to fig. 35 except for the reflector and the position of the lampboard.)
5. Encrypt "bed" using the Enigma-like device of fig. 37 (note the relationship of fig. 37 to figs. 35 & 36.)
6. Verify or correct the number of keys given on p. 136.

<u>Chap. 4, see pgs. 157-8</u>

1. Check that in 1938 the number of scrambler arrangements becomes 60 -- up from 6.
2. Check that with the new plugboard too, the # of keys is now 159×10^{18}.

<u>Ideas, People, Events and Dates</u>

<u>Chap. 3:</u> Cipher machines, Alberti's rotating disks (15th Cent.), Arthur Scherbius, Enigma (1918 thru WWII), keyboard (input), plugboard, scramblers=rotors, reflector, lampboard)(output), formula for counting pairs: n(n-1)/2, counting keys, American Black Chamber (WWI thru 1920's), Herbert

Yardley, Winston Churchill, Britain's Room 40.

Chap.4: 1926 Enigma messaging starts, Poland's Biuro Szyfrow, Berlin's Chiffrierstelle, Hans-Thilo Schmidt, Marian Rejewski, bombes, 3--->5 scramblers in 3 positions in 60 ways, plugboard cables 6 pairs----> 10 pairs, counting # of ways,, Blitzkrieg starts 1939, Hitler invades Poland, Poland's Enigma work smuggled to Britain.

Bletchley Park 1939 thru WWII, cillies (girlfriends' initials, keyboard triples), cribs, rules restricting rotor settings. Alan Turing, undecidability in Math, Turing Machines, improved bombs. Ultra, D-Day 1944, debt to codebreakers remains secret till early 70's, Turing persecuted for his homosexuality, suicide.

Chap. 5: Typex, Sigaba, Purple ciphers of WWII, Navajo code talkers, Navajo code lexicon. Ancient Egypt's writing systems: hieroglyphics, hieratic, demotic; 3000BC - 450AD. Coptic spoken & written. A. Kircher (1652). Decrypting 1798 to 1832: Napoleon, Thomas Young, J. F. Champollion. Phonogram vs semagram, cartouche, Rosetta Stone.