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Scope Economies in Banking: The Hybrid Box-Cox Function

*A. Sinan Cebenoyan**

Abstract

This paper provides evidence on scope economies in banking using the Box-Cox hybrid functional form on five products and two inputs. A factor demand equation augmented system of equations is estimated using the iterative Zellner technique. Strict Willig-type scope estimates indicate that there are scope diseconomies in the sample of banks in the Functional Cost Analysis (FCA) Program data for 1983.

Introduction

Given the recent turbulent market conditions and the general financial environment with its ongoing deregulation, it becomes imperative that we understand the cost structure and its implications for market structure in banking. Most studies to date have focused on the estimation of scale economies (economies due to size) and scope economies (economies due to joint production) in banking as a guide to possible exploitable efficiencies in the cost structure. To name a notable few, Benston et al. [2]; Berger, Hanweck, and Humphrey [3]; Cebenoyan [6]; Clark [8]; Gilligan, Smirlock, and Marshall [10]; Kim [11]; Kolari and Zardkoohi [12]; Mester [13]; and Murray and White [14] have provided us with estimates of scale economies and sometimes scope economies for financial institutions.

Transcendental logarithmic cost functions have mostly been used to estimate the output elasticity of total costs, thus economies of scale. The translog function proved to be good in estimating scale economies. The gen-

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eral indication is that scale economies are exhausted beyond small output levels—usually less than 100 million dollars of deposits. (Cebenoyan [6] provides evidence for scale economies in larger banks for part of his sample.) The evidence can best be said to be mixed with regards to scale economies.

For scope economies, the literature does not provide an acceptable strict test. The problem, as most authors acknowledge, is that the translog function cannot provide a strict test for scope economies—it is undefined at zero output levels. Attempts to circumvent the shortcomings of the translog have only produced approximate, mostly inappropriate, and generally incomplete results that are subject to errors. Our understanding of the cost structure in banking is thus incomplete. Any discussion of comparisons of costs of financial supermarkets versus financial boutiques is pointless if we cannot test for the existence of scope economies. This study attempts to provide that test.

We introduce the Box-Cox transformations (see Box and Cox [4]) into the well-known translog cost functions to provide possibilities for proper testing of scope economies in multiproduct banking firms. The Box-Cox metric promises to overcome a major drawback of the translog functional form: it allows zero output levels. Previous researchers unsuccessfully attempted to circumvent the shortcomings of the translog form (Benston et al. [2]; Gilligan, Smirlock, and Marshall [10]; Murray and White [14]; Kim [11]; among others) by estimating the function at some close-to-zero output level. These efforts proved less than satisfactory because the results could have changed for any other approximation. By focusing on subadditivity in the production process, Mester [13] and Berger, Hanweck, and Humphrey [3] conclude that there are no scope economies in savings and loans [13] nor in commercial banks [3]. Both studies use the translog function, thus they do not provide a strict test for scope economies. We use the same data and time period as the Cebenoyan [6] and Berger, Hanweck, and Humphrey [3] studies on commercial banks for comparability as well as for consistency of databases, and we provide the missing strict tests of scope economies for unit and branch banks.

The paper is organized as follows. The first section discusses the Box-Cox functional form. The next section describes the data and variables. The third section gives the results. The last section offers some conclusions.

The Box-Cox Metric and the Hybrid Cost Function

If it is agreed that joint production and economies of scope are important in banking, then we must abandon the translog function in its present form and move to a more general functional form that will allow estimation of economies of scope. When searching for a more general functional form, we do not want to part with the desirable properties of the translog function entirely. To maintain homogeneity of degree one in input prices, we use the log metric on prices and outputs in our cost function and impose parametric restrictions on it. However, as Caves, Christensen, and Tretheway [5] report, the use of the log metric for the outputs is unnecessary for this homogeneity property, and it prevents the use of any observations with zero output values. It would, therefore, be quite convenient to maintain the log metric for total costs and input prices but choose a different metric for outputs that is well defined for zero values.

The Box-Cox [3] metric is one that not only permits zero values but also contains the natural log metric as a limiting case. (A similar version was used by Clark [8], but Clark used a Cobb-Douglas underlying functional form transforming all the variables and did not consider the existence of scope economies.) The Box-Cox metric is

$$\begin{aligned} f_i(Y_i) &= (Y_i^\lambda - 1)/\lambda \qquad \text{for } \lambda \neq 0 \\ f_i(Y_i) &= \ln Y_i \qquad \text{for } \lambda = 0 \end{aligned} \tag{1}$$

Provided that λ is strictly positive, the Box-Cox metric is well defined for zero output levels:

$$f_i(0) = -1/\lambda.$$

Following Caves, Christensen, and Tretheway [5], we form a general quadratic flexible form using the natural log as the metric for total costs and input prices and the Box-Cox metric for output quantities. The resulting multicost function does not have the flaws that we had observed in other functional forms. Thus we write

$$\begin{aligned}\ln C = \alpha_0 &+ \sum_i \alpha_i (Y_i^\lambda - 1)/\lambda \\ &+ \sum_k \beta_k \ln P_k + 1/2 \sum_k \sum_l \gamma_{kl} \ln P_k \ln P_l \\ &+ 1/2 \sum_i \sum_j \delta_{ij} (Y_i^\lambda - 1)/\lambda \, (Y_j^\lambda - 1)/\lambda \\ &+ \sum_i \sum_k \rho_{ik} \, (Y_i^\lambda - 1)/\lambda \ln P_k \qquad (2)\end{aligned}$$

where $\delta_{ij} = \delta_{ji}$ and $\gamma_{kl} = \gamma_{lk}$ for symmetry. Since every cost function must exhibit homogeneity of degree one in factor prices in order to correspond to a well-behaved production function, we impose the following linear restrictions on (2):

$$\begin{aligned}&\sum_k \beta_k = 1 \qquad (k = 1, 2, \ldots, n); \\ &\sum_k \gamma_{kl} = 0 \qquad (l = 1, 2, \ldots, n); \\ &\sum_k \rho_{ik} = 0 \qquad (i = 1, 2, \ldots, m). \qquad (3)\end{aligned}$$

We will use Christensen and Greene's [7] method of augmenting our equations by including the factor share equations (Diewert [9]) using Shephard's lemma. These additional equations increase the efficiency of the estimates without introducing any more unknowns. The following factor share equations will now apply:

$$\begin{aligned}\partial \ln C/\partial \ln P_k &= W_k \\ &= \beta_k + \sum_l \gamma_{kl} \ln P_l + \sum_i \rho_{ik} (Y_i^\lambda - 1)/\lambda. \qquad (4)\end{aligned}$$

We will extend our analysis to more than the usual two products, namely deposits and loans, to more finely break down the two and test for economies of scope. Economies of scope are said to exist in a two-product case at point Y if and only if

$$C(Y_1, Y_2) < C(Y_1, 0) + C(0, Y_2). \qquad (5)$$

Thus by constraining the function to exhibit zero levels of output, we can calculate economies of scope. We will use a measure developed by Willig [15] to estimate the degree of scope economies.

The Sample

Data

As noted earlier, we use the same database used in some recent studies: the Functional Cost Analysis (FCA) Program of the Federal Reserve Bank of New York for 1983. The FCA data have well-known drawbacks, such as including only small banks, not being able to identify nor geographically locate banks that participate in the program, etc. We will differentiate between branch and unit banks by splitting our sample into two groups and conducting a parallel analysis.

The evidence does not indicate that there are significant differences between production and intermediation approaches to bank production. We will use the production approach and define our variables as follows.

Variables

Total operating costs (TC). Costs are measured as the sum of labor costs and capital expenditures, as used in the input cost measures of the bank defined below.

Bank output (DD, TD, RL, IL, and CL). We follow the production approach and use the annual average number of demand deposit (*DD*), time deposit (*TD*), real estate loan (*RL*), installment loan (*IL*), and commercial and other loan (*CL*) accounts serviced as our measures of output.

Factor prices (Cap and Wage). We assume that capital and labor are the two factors of bank production. The cost of labor, *Wage*, is measured as average annual salary plus fringe benefits and fees paid to all employees of the bank. The price of capital, *Cap*, will have to be taken from the FCA data since we cannot identify banks and cannot substitute geographical proxies such as rental costs as some researchers were able to do (Berger, Hanweck, and Humphrey [3]). We estimate the cost of capital as the ratio of the sum of furniture and equipment, occupancy, and computer (including software) expenses to the building and equipment less depreciation items from the balance sheet. This gives us a proxy for the cost of capital as dollar expenditures per dollar of investment in real assets.

Estimation and Results

As defined in the previous section, we have five products and two inputs in our cost function. To increase the efficiency of the parameters, we invoked Shephard's lemma and introduced the labor share equation. This resulted in a system of equations consisting of equations (2) and (4) that was estimated using the iterative Zellner [16] technique.

This flexible functional form takes its shape from the data. Thus, for each sample of unit and branch banks, the value of λ must be estimated through a time-consuming iterative search. We allowed λ to start from 0.9 and reduced it by 0.1 in each pass. We recorded the log determinant of the residuals at each pass. When we reached a minimum, we continued to reduce λ further to make sure it was a global minimum. Once that was established, we had our estimates for that particular sample. We report our results in Tables 1 and 2.

As Baumol [1] demonstrates, for monopoly to be the least costly form of productive organization, the critical concept is strict subadditivity of the cost function, meaning that the cost of the sum of any m output vectors is less than the sum of the costs of producing them separately. Economies of scope are necessary for subadditivity; therefore, measuring it properly is of crucial importance.

Measurement of scope economies, as explained earlier, has been unsuccessfully attempted by a number of authors. To measure economies of scope, the underlying cost function must be defined at zero levels of product quantities and must also exhibit the desirable properties of the translog form. Our function does just that. After Willig [15], we measure the degree of economies of scope as

$$\begin{aligned} SCP = {} & [C(DD,0,0,0,0) + C(0,TD,0,0,0) \\ & + C(0,0,RL,0,0) + C(0,0,0,IL,0) \\ & + C(0,0,0,0,CL) - C(DD,TD,RL,IL,CL)]/ \\ & C(DD,TD,RL,IL,CL). \end{aligned} \tag{6}$$

TABLE 1

Box-Cox Coefficients for Unit Banks, λ = 0.7
(*t*-ratios in parentheses)

Variable	Coefficient	Variable	Coefficient
Constant	7.93388 (18.223)	*TDIL*	.0000006 (1.780)
DD	.004748 (11.648)	*TDCL*	−.000002 (−2.047)
TD	.000004 (.014)	*RLIL*	−.000003 (−1.131)
RL	.001309 (1.069)	*RLCL*	−.000003 (−.808)
IL	−.000082 (−.144)	*ILCL*	−.0000001 (−.081)
CL	.000901 (1.027)	ln *WAGE2*	.051167 (7.344)
ln *CAP*	.833413 (10.879)	ln *CAP2*	.051167 (7.344)
ln *WAGE*	.166587 (2.175)	ln *WLNC*	−.051167 (−7.344)
DD2	−.000004 (−5.072)	*DDK*	.000070 (3.174)
TD2	−.0000004 (−1.565)	*DDW*	−.000070 (−3.174)
RL2	−.000003 (−.738)	*TDK*	−.000030 (−2.132)
IL2	−.000002 (−1.398)	*TDW*	.000030 (2.132)
CL2	−.000007 (−1.847)	*RLK*	.000002 (.043)
DDTD	−.0000001 (.255)	*RLW*	−.000002 (−.043)
DDRL	.0000007 (.403)	*ILK*	−.000005 (−.177)
DDIL	.00000089 (1.000)	*ILW*	−.000005 (.177)
DDCL	.000005 (3.607)	*CLK*	−.000022 (−.437)
TDRL	.0000009 (.821)	*CLW*	.000022 (.437)

This is the proportion by which costs would increase if a multiproduct firm were split along product lines.

Each component of equation (6) was computed by multiplying the estimated coefficients of the applicable regression with the vector of means of the variables. For

TABLE 2

Box-Cox Coefficients for Branch Banks, λ = 0.3
(*t*-ratios in parentheses)

Variable	Coefficient	Variable	Coefficient
Constant	6.27194 (20.956)	*TDIL*	−.000666 (−3.356)
DD	.068074 (9.017)	*TDCL*	.000348 (2.484)
TD	.011241 (1.773)	*RLIL*	.000446 (1.909)
RL	.037859 (5.445)	*RLCL*	−.000398 (−2.485)
IL	.024486 (3.357)	*ILCL*	.000116 (.723)
CL	−.005246 (−.876)	ln *WAGE*2	.046237 (10.511)
ln *CAP*	.784276 (16.192)	ln *CAP*2	.046237 (10.511)
ln *WAGE*	.215724 (4.454)	ln *WLNC*	−.046237 (−10.511)
*DD*2	−.000166 (−.635)	*DDK*	.000206 (.640)
*TD*2	.000234 (.956)	*DDW*	−.000206 (−.640)
*RL*2	.000558 (2.191)	*TDK*	−.000429 (−1.511)
*IL*2	.000647 (2.955)	*TDW*	.000429 (1.511)
*CL*2	−.000094 (−.478)	*RLK*	.001359 (4.326)
DDTD	.000019 (.124)	*RLW*	−.001359 (−4.326)
DDRL	−.000303 (−1.669)	*ILK*	−.000273 (−.851)
DDIL	−.000211 (−.982)	*ILW*	.000273 (.851)
DDCL	−.000123 (−.701)	*CLK*	−.000145 (−.540)
TDRL	−.000304 (−1.276)	*CLW*	.000145 (.540)

zero output levels, we used $-1/\lambda$ as the mean of the product quantity. Since the function computes total costs in logarithms, we took the exponential of each of the above components and calculated *SCP*.

Estimates of Degrees
of Scope Economies *SCP*

λ	Unit	λ	Branch
.7	−0.28	.3	−0.67

The results indicate that there are strong diseconomies of scope in banking, both in the unit as well as in the branch level. The results support the earlier studies that found evidence of diseconomies of scope in multiproduct banking firms. There appear to be disadvantages to supermarkets of financial services in costs. As deregulation proceeds, there should be little tendency towards a consolidation of market participants—there is ample room for the specialized banking firms.

It can be argued that the above results are prone to extrapolation errors since "they require evaluation of a cost function at (or near) zero outputs which are generally far outside the sample over which the cost function is estimated" (Berger, Hanweck, and Humphrey [3], p. 503). Although it is true that many banks produced all of the outputs at some positive level, there were about 70 banks that produced zero units of one or more of the five outputs. Furthermore, even if a handful of banks appear to produce zero output levels, it would be incorrect to dismiss them entirely and avoid a strict scope test. Subadditivity studies will be incomplete without properly establishing the necessary conditions for subadditivity.

It must also be acknowledged that the FCA data fall short of representing the whole banking sector. The results are sample sensitive and should only be generalized to the banking population to the extent that the population is represented by the FCA sample. The literature has generally made use of the FCA data, and this study attempts to complete the work.

Conclusion

This paper provides evidence on scope economies in banking using the multiproduct Box-Cox hybrid cost function. The use of the Box-Cox metric allowed us to develop strict tests for scope economies. Previous research has tried to circumvent the shortcomings of the translog form by either approximately estimating scope economies or by skipping over the translog form and estimating subadditivity directly. Approximations are inconclusive in nature, and subadditivity requires scope economies as a necessary condition. The results imply that the necessary conditions for subadditivity are not met in bank cost functions.

Acknowledging the shortcomings of the FCA data, we refrain from declaring these results representative of the whole banking sector. But we believe there is a strong indication that as banking deregulation proceeds, the number of banking firms operating will not diminish significantly. There is no evidence indicating monopolistic tendencies in banking markets.

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